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IMPORTANT REMARKS ON THE PROBLEM OF NEUTRINO PASSING THROUGH THE MATTER

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Важные замечания к проблеме прохождения нейтрино
через вещество

Считается, что при прохождении нейтрино через вещество возникает резонансное усиление осцилляций нейтрино. Показано, что уравнение Вольфенштейна, описывающее прохождение нейтрино через вещество, имеет недостаток (не учитывает закон сохранения импульса, т. е. предполагается, что энергия нейтрино в веществе изменяется, а импульс остается неизменным). Это приводит, например, к изменению эффективной массы нейтрино на величину $0{,}87 \cdot 10^{-2}$ эВ, которая возникает от мизерной энергии поляризации вещества, равной $5 \cdot 10^{-12}$ эВ. После устранения этого недостатка, т. е. после учета изменения импульса нейтрино в веществе, получены решения данного уравнения. В этих решениях возникают очень маленькие усиления осцилляции нейтрино в солнечном веществе из-за маленького значения энергии поляризации вещества при прохождении нейтрино. Также получены решения этого уравнения для двух предельных случаев.

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Important Remarks on the Problem of Neutrino Passing
through the Matter

It is supposed that resonance enhancement of neutrino oscillations in the matter appears while a neutrino is passing through the matter. It is shown that Wolfenstein's equation, for a neutrino passing through the matter, has a disadvantage (it does not take into account the law of momentum conservation; i.e., it is supposed that in the matter the neutrino energy changes, but its momentum does not). It leads, for example, to changing the effective mass of the neutrino by the value $0.87 \cdot 10^{-2}$ eV from a very small value of energy polarization of the matter caused by the neutrino, which is equal to $5 \cdot 10^{-12}$ eV. After removing this disadvantage (i.e., taking into account that neutrino momentum also changes in matter) we have obtained a solution to this equation. In this solution a very small enhancement of neutrino oscillations in the solar matter appears due to the smallness of the energy polarization of the matter caused by the neutrino. Two possible solutions to this equation are also given for the limiting cases.

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1. INTRODUCTION

The suggestion that, in analogy with $K^0, \bar{K}^0$ oscillations, there could be neutrino–antineutrino oscillations ($\nu \to \bar{\nu}$) was made by Pontecorvo [1] in 1957. It was subsequently considered by Maki et al. [2] and Pontecorvo [3] that there could be mixings (and oscillations) of neutrinos of different flavors (i.e., $\nu_e \to \nu_\mu$ transitions).

The first experiment [4] on the solar neutrinos has shown that there is a deficit of neutrinos; i.e., the solar neutrinos flux detected in the experiment was few times smaller than the flux computed in the framework of the Sun Standard Model [5]. The subsequent experiments and theoretical computation have confirmed the deficit of the solar neutrinos [6].

The short base reactor and accelerator experiments [7] have shown that there is no neutrino deficit, then this result was interpreted as an indication that the neutrino vacuum mixing angle is very small (subsequent experiments have shown [8] that this vacuum angle is big and near the maximal value). Then the question arises: what is the deficit of the solar neutrinos related with? In 1978 the work by L. Wolfenstein [9] appeared where an equation describing a neutrino passing through the matter was formulated (afterwards that equation was named Wolfenstein's). In the framework of this equation the enhancement of neutrino oscillations in the matter arises via weak interactions (for critical remarks on this equation, see [10]). This mechanism of neutrino oscillations enhancement in the matter attracted the attention of neutrino physicists after publications [11] by S. P. Mikheyev and A. Ju. Smirnov where it was shown that in the framework of this equation the resonance enhancement of neutrino oscillations in the matter would take place. Also it is clear that the adiabatic neutrino transitions can arise in the matter if effective masses of neutrinos change in the matter [12].

This work is devoted to discussion of neutrino oscillations in the matter by using the Wolfenstein-type equation.

2. THE NEUTRINO (PARTICLE) PASSING THROUGH THE MATTER

Before consideration of a neutrino (particle) passing through the matter, it is necessary to gain some understanding of the physical origin of this mechanism. While the neutrino is passing through the matter there can proceed two processes — neutrino scattering and polarization of the matter by the neutrino. Our interest is related with neutrino elastic interactions in the matter, namely with neutrino forward elastic scattering, i.e., polarization of the matter by the passing neutrino. The neutrino passing through the matter at its forward scattering can be considered by using the following Wolfenstein's equation [9]:

$$i\frac{d\nu_{\rm ph}}{dt} = (\hat{E} + \hat{W})\nu_{\rm ph} \equiv \left(\sqrt{p^2 I + \hat{M}^2} + \hat{W}\right)\nu_{\rm ph}, \tag{1}$$

where $p, \hat{M}^2, \hat{W}_i$ are, respectively, the momentum, the (nondiagonal) square mass matrix in vacuum, and the matrix, taking into account neutrino interactions in the matter,

$$\begin{gathered}\nu_{\rm ph} = \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}, \quad \hat{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \\ \hat{M}^2 = \begin{pmatrix} m^2_{\nu_e\nu_e} & m^2_{\nu_e\nu_\mu} \\ m^2_{\nu_\mu\nu_e} & m^2_{\nu_\mu\nu_\mu} \end{pmatrix}, \quad \hat{W} = \begin{pmatrix} \hat{W}_e & 0 \\ 0 & \hat{W}_\mu \end{pmatrix}.\end{gathered} \tag{2}$$

This equation has a standard solution found by S. P. Mikheyev and A. Ju. Smirnov [11] which leads to resonance enhancement of neutrino oscillations in the matter. We consider this solution below.

2.1. The Standard Mechanism of Resonance Enhancement of Neutrino Oscillations in the Matter and Some Critical Remarks. In the ultrarelativistic limit $\left(E \simeq p\hat{I} + \frac{\hat{M}^2}{2p}\right)$, the evolution equation for the neutrino wave function $\nu_{\rm ph}$ in the matter has the following form [9, 11]:

$$i\frac{d\nu_{\rm ph}}{dt} = \left(p\hat{I} + \frac{\hat{M}^2}{2p} + \hat{W}\right)\nu_{\rm ph}. \tag{3}$$

Since the neutrino has relativistic velocity, taking into account that at obtaining Eq. (1) it is supposed that the neutrino momentum in the matter does not change (then the neutrino mass has to change), we can write the expression for neutrino energy $\hat{E}' = \hat{E} + \hat{W}$ in the matter in the following form:

$$\hat{E}' = \sqrt{p^2 + \hat{M}'^2} \simeq p + \frac{\hat{M}'^2}{2p},$$

hence $\hat{M}'^2 = \hat{M}^2 + 2p\hat{W}$. The expression for W is $W = \sqrt{2}G_F n_e$ [9, 11].

If we suppose that neutrinos in the matter behave analogously to the photon in the matter (i.e., the polarization appears while the neutrino is passing through the matter) and the neutrino refraction indices are defined by the following expression (in case of antineutrino $W < 0$ and therefore it is of no interest since in this case $n < 1$ and the resonance effect cannot arise):

$$n_i = 1 + \frac{2\pi N}{p^2} \mathrm{Re}\, f_i(0) = 1 + 2\frac{\pi W_i}{p}, \tag{4}$$

where i is a type of neutrinos (e, μ, τ), N is density of the matter, $f_i(0)$ is a real part of the forward scattering amplitude, then W_i characterizes the polarization of the matter by neutrinos (i.e., it is the energy of the matter polarization). In reality, as we will see below, there is a fundamental difference: the photon is a massless particle, while the neutrino is a massive particle, and this distinction is basic.

The electron neutrino (ν_e) in the matter interacts via $W^{\pm}, Z^0$ bosons and ν_μ, ν_τ interact only via Z^0 boson. These differences in interactions lead to the following differences in the refraction coefficients of ν_e and ν_μ, ν_τ:

$$\begin{gathered} \Delta n = \frac{2\pi N_e}{p^2} \Delta f(0), \\ \Delta f(0) = \sqrt{2} \frac{G_F}{2\pi} p, \\ E_{\text{eff}} = \sqrt{p^2 + m^2} + \langle e\nu | H_{\text{eff}} | e\nu \rangle \approx p + \frac{m^2}{2p} + \sqrt{2} G_F N_e, \end{gathered} \tag{5}$$

where G_F is the Fermi constant.

The energy of the matter polarization E is

$$E \approx W = \sqrt{2} G_F N_e, \quad W = 7.6 \left(\frac{N_e}{n_0} \right) \cdot 10^{-14} \text{ eV}, \tag{6}$$

where N_e is the electron density in the matter, n_0 is Avogadro number. For the Sun

$$E^{\text{Sun}} \approx 10^{-13}\text{–}10^{-11} \text{ eV}. \tag{7}$$

Therefore, the velocities (or effective masses) of ν_e and ν_μ, ν_τ in the matter are different. And at the suitable density of the matter this difference can result in resonance enhancement of neutrino oscillations in the matter [9, 12]. The expression for $\sin^2 2\theta_m$ in the matter has the following form:

$$\sin^2 2\theta_m = \sin^2 2\theta \left[\left(\cos 2\theta - \frac{L_0}{L^0} \right)^2 + \sin^2 2\theta \right]^{-1}, \tag{8}$$

where $\sin^2 2\theta_m$ and $\sin^2 2\theta$ characterize neutrino mixings in the matter and vacuum, L_0 and L^0 are lengths of neutrino oscillations in vacuum and neutrino refraction length in the matter:

$$L_0 = \frac{4\pi E_\nu \hbar}{\Delta m^2 c^3}, \quad L^0 = \frac{\sqrt{2}\pi\hbar c}{G_F N_e}, \tag{9}$$

where E_ν is neutrino energy, $\Delta m^2 = m_2^2 - m_1^2$ is difference between squared neutrino masses, c is light velocity, $\hbar$ is Planck constant.

The probability of $\nu_e \to \nu_\mu$ neutrino transitions is given by the following expression ($E \simeq pc$):

$$P(E, t, \ldots) = 1 - \sin^2 2\theta_m \sin^2 \frac{2\pi ct}{L_m}, \tag{10}$$

where $L_m = \dfrac{\sin 2\theta_m}{\sin 2\theta} L_0$.

At resonance

$$\cos 2\theta \cong \frac{L_0}{L^0}, \quad \sin^2 2\theta_m \cong 1, \quad \theta_m \cong \frac{\pi}{4}. \tag{11}$$

The expression (11) for resonance condition can be rewritten in the following form:

$$\sqrt{2} G_F N_e = \frac{\Delta m^2}{2E_{\nu_e}^{\mathrm{res}}} \cos 2\theta, \tag{12}$$

or

$$E_\nu^{\mathrm{res}} = \frac{\Delta m^2 \cos 2\theta}{2W} \to \Delta m^2 - \frac{2E_\nu^{\mathrm{res}} W}{\cos 2\theta} = 0. \tag{13}$$

If we consider $\nu_e \to \nu_\mu$ and use KamLAND data [13]

$$\begin{gathered} \tan^2\theta_{12} = 0.56(+0.10, -0.07)(\text{stat.})(+0.1, -0.06)(\text{syst.}), \quad \theta = 36.8^\circ, \\ \Delta m_{12}^2 = 7.58(+0.14, -0.13)(\text{stat.}) \pm 0.15(\text{syst.}) \cdot 10^{-5}\ \text{eV}^2, \end{gathered} \tag{14}$$

then at $n_e = 65.8 n_0$ the energy W^{Sun} of neutrino polarization is $W^{\mathrm{Sun}} = 5 \cdot 10^{-12}$ eV and for E_ν^{res} we obtain

$$E_\nu^{\mathrm{res}} = 2.14 \cdot 10^6\ \text{eV} = 2.14\ \text{MeV}. \tag{15}$$

The expressions (11)–(15) mean that when the electron neutrino with energy $E_\nu = 2.14$ MeV is passing through the Sun matter, the effective mass of the electron neutrino becomes equal to the muon neutrino mass (see below the expressions (16) and (17)) and as a result there is a resonance transition of electron neutrinos into muon neutrinos. In this case the change in the squared mass of neutrino ν_1 is

$\Delta m_{12}^2 = 7.58 \cdot 10^{-5}$ eV^2 (we suppose that $m_2 > m_1$), i.e., the effective mass of neutrino ν_1 is

$$m_{1,\mathrm{eff}}^2 \simeq m_1^2 + 7.58 \cdot 10^{-5} \text{ eV}^2, \tag{16}$$

and (in reality $m_{\nu_e}^{\mathrm{matt}} \simeq m_{\nu_\mu}$ [10])

$$m_{1,\mathrm{matt}}^2 \approx m_2^2. \tag{17}$$

We see that this additional big mass arises at polarization of the matter by the neutrino with energy $W = 5 \cdot 10^{-12}$ eV. It is a very strange result! A primary ultrarelativistic electron neutrino having energy $E_\nu = 2.14 \cdot 10^6$ eV interacts with the matter with the energy $W = 5 \cdot 10^{-12}$ eV and as a result we obtain the mass increase by $\delta m \approx \sqrt{7.58 \cdot 10^{-5}} = 0.87 \cdot 10^{-2}$ eV. We know that the matter polarization has to move with the velocity equal to that of the neutrino which generates this polarization. Then the energy of the electron neutrino has to increase by

$$\Delta E_\nu \approx \delta m \gamma, \tag{18}$$

where $\gamma = \dfrac{E_\nu}{m_\nu}$ and neutrino velocity $v \simeq c$. Why have we arrived at this result when solving this equation? It is a consequence of the above approach when we included the full energy of the matter polarization in the neutrino mass. It is possible only at a serious violation of the law of energy-momentum conservation. We can suppose that increase in the neutrino effective mass is accompanied by decrease in the neutrino velocity; i.e., the energy is conserved, but it does not save the situation since the mass increase is

$$\delta m \approx \sqrt{7.58 \cdot 10^{-5}} = 0.87 \cdot 10^{-2} \text{ eV}, \tag{19}$$

while the energy of matter polarization is $W = 5 \cdot 10^{-12}$ eV. It means that the mechanism of resonance enhancement of neutrino oscillations in the matter can be realized at violation of the law of energy-momentum conservation.

Now we consider a general case where the total energy of the matter polarization is included in the neutrino kinetic energy and mass.

2.2. The General Case of the Neutrino Passing through the Matter. In [14] (Dirac's theory of direct interaction) a general method was developed to avoid the paradox similar to the above one when a huge mass change arises from very small energy. The example we consider is very simple; therefore, it will be sufficient to take into account the law of momentum conservation besides the law of energy conservation.

Above we have considered the case when the full energy of the matter polarization caused by the neutrino is included in the mass. If the particle (neutrino) interaction with the matter is the left-right symmetric one, then the mass can be generated there (as in strong and electromagnetic interactions). In this case we have to share the full energy of the matter polarization caused by the particle (neutrino) between the kinetic and mass parts of the particle (neutrino) energy. We will suppose that the weak interactions are the left-right symmetric ones and then we will not consider the problem of mass generation in the weak interactions.

To solve this problem, it is necessary to compute the full energy W of the matter polarization and then, taking into account the law of energy-momentum conservation in the vacuum (p, M) and in the matter (p', M'), to distribute this full energy of polarization between the kinetic and mass parts of the particle (neutrino) energy. It coincides with the problem of polaron for a certain interaction (for references see Wikipedia). So in the matter

$$E' = E + W, \tag{20}$$

and $(p_W = W v_\nu)$

$$p' = p + p_W, \tag{21}$$

since $p^2 \gg M^2$ the neutrino is an ultrarelativistic particle and $v_\nu \simeq c$ (c is the light velocity), then with $p_W \simeq W$

$$p' \simeq p + W. \tag{22}$$

The expression for neutrino energy E' in the matter (after taking into account the expression (22)) will then have the following form:

$$\sqrt{p^2 + M^2} + W = \sqrt{p'^2 + M'^2} \rightarrow p^2 + M^2 + 2W\sqrt{p^2 + M^2} + W^2 = \\ = M'^2 + p'^2 \equiv M'^2 + p^2 + 2pW + W^2, \tag{23}$$

where $p' \simeq p + W$. Then using the expressions (20), (22) and taking into account that $p^2 \gg M^2$, from the expression (23) we obtain

$$M'^2 - M^2 \simeq Wp\left(\frac{M^2}{p^2}\right) \rightarrow M'^2 \simeq M^2 + Wp\left(\frac{M^2}{p^2}\right). \tag{24}$$

If one takes into account that $p^2 \gg M^2$ and $W \approx 10^{-12}$, then

$$M'^2 \simeq M^2 + Wp\left(\frac{M^2}{p^2}\right) \simeq M^2. \tag{25}$$

In this case Wolfenstein's equation has the same form as Eq. (1) since the term W originated from the left-right symmetric interaction is inserted into this equation with the left-right symmetric wave function

$$i\frac{d\nu_{\rm ph}}{dt} = (\sqrt{p'^2 + M'^2}\nu_{\rm ph}), \tag{26}$$

then using that $\sqrt{p'^2 + M'^2} \simeq \left(p'\hat{I} + \frac{\hat{M}'^2}{2p'}\right)$ and the expression (24), we obtain

$$i\frac{d\nu_{\rm ph}}{dt} = \left(p'\hat{I} + \frac{\hat{M}^2 + Wp\left(\frac{M^2}{p^2}\right)}{2p'}\right)\nu_{\rm ph}, \tag{27}$$

or taking into account the expression (25) or that the term $Wp\left(\frac{M^2}{p^2}\right)$ is very small, we come to the following expression:

$$i\frac{d\nu_{\rm ph}}{dt} = \left(p'I + \frac{M^2}{2p'}\right)\nu_{\rm ph}, \tag{27$'$}$$

where $p' = (p + W)$. The expression for the neutrino transition probability in this case has the form

$$P(E', t, \ldots) = 1 - \sin^2 2\theta' \sin^2 \frac{\pi ct}{L''_0}, \tag{28}$$

where $E' \simeq p'c$ and $L''_0 = \frac{\sin 2\theta'}{\sin 2\theta}L'_0$; since $M'^2 \simeq M^2$, one obtains $\sin\theta \simeq \sin\theta'$, $L''_0 \simeq L'_0$:

$$L'_0 = \frac{4\pi E'_\nu \hbar}{\Delta m^2 c^3}, \quad \sin^2 2\theta' \simeq \sin^2 2\theta. \tag{29}$$

So, since the change in the neutrino effective mass is very small, the change in the neutrino transition probability arises only owing to the neutrino momentum change. It is necessary to take into account that $p \gg W$, then this change will also be very small. We have arrived at the following conclusion: Taking into account not only the law of neutrino energy conservation but also the law of neutrino momentum conservation, the neutrino transition probability in the matter change is very small and no noticeable enhancement of the neutrino oscillations in the solar matter appear (i.e., the condition (17) cannot be fulfilled). We see that the term which generates the huge change in the neutrino effective mass and leads to the resonance enhancement of neutrino oscillations in the solar matter appears since the law of momentum conservation was not taken into account in the original Wolfenstein's equation (as a matter of fact it is necessary to include this term in the neutrino momentum, but not in the neutrino mass).

However, it is necessary to remark that at very high matter densities (for example, in super new stars) the term $Wp\frac{M^2}{p^2}$ can lead to resonance enhancements of neutrino oscillations.

2.3. Other Solutions to Wolfenstein's Equation. Besides the above solution to Wolfenstein's equation, there can be two particular solutions. Now turn to their consideration.

2.3.1. The First Case (Nonrelativistic Case). Consider the case when the full energy of matter polarization is included in the neutrino mass. The expression for neutrino energy will then have the form

$$\sqrt{p^2 + M^2} + W = \sqrt{p^2 + M'^2} \to M'^2 = M^2 + 2W\sqrt{p^2 + M^2} + W^2; \quad (30)$$

taking into account that $p^2 \gg M^2$, $p^2 \gg W^2$, we obtain

$$M'^2 \simeq M^2 + 2Wp. \quad (31)$$

Wolfenstein's equation can then be written in the form

$$i\frac{d\nu_{\rm ph}}{dt} = \sqrt{p^2 I + M'^2}\nu_{\rm ph} \to i\frac{d\nu_{\rm ph}}{dt} = \left(p + \frac{M^2 + 2Wp}{2p}\right)\nu_{\rm ph}; \quad (32)$$

i.e., we exactly come to Eq. (1). But as is shown above, in this equation the law of momentum conservation is violated. Then the following question arises: Under what condition can such a type of equation be realized? It is clear that the approach used should work in a nonrelativistic case when $p^2 \ll M^2$, but not in the ultrarelativistic case.

The expression for neutrino energy will then have the following form:

$$\sqrt{p^2 + M^2} + W = \sqrt{p^2 + M'^2} \to M'^2 = M^2 + 2W\sqrt{p^2 + M^2} + W^2. \quad (33)$$

Taking into account that $p^2 \ll M^2$, $W^2 \ll M^2$, we obtain

$$M'^2 \simeq (M + W)^2. \quad (34)$$

By using the expression (34) Wolfenstein's equation (1) can be written as

$$i\frac{d\nu_{\rm ph}}{dt} = \sqrt{p^2 I + M'^2}\nu_{\rm ph} \to i\frac{d\nu_{\rm ph}}{dt} = \left(\hat{M}' + \frac{\hat{p}^2}{2M'^2}\right)\nu_{\rm ph}, \quad (35)$$

where M'^2 is given by the expression (35).

We see that at low energies we can include the full energy W of the matter polarization in the neutrino mass and the expressions (8)–(17) will be replaced by the expressions (33)–(35). By diagonalization of the mass matrix M' we then obtain that the neutrino mixing angle changes and under appropriate conditions (at enough large value of W) the resonance enhancement of neutrino oscillations in the matter will take place. It is necessary to remark that the full energy of solar matter polarization $E^{\rm Sun} \approx 10^{-13}$–$10^{-11}$ eV, which arises in the passage of the electron neutrino through the Sun, is too small to generate the resonance enhancement of neutrino oscillations. Above we supposed that masses can be generated in the framework of standard weak interactions, i.e., there is no problem with mass generation.

As was stressed above, this approach becomes physically realizable only when $p^2 \ll M^2$ and then the full energy of the matter polarization is transformed into the neutrino mass, but we have to keep in mind that the neutrinos produced in weak interactions are ultrarelativistic since the neutrino masses are very small. So, since the solar neutrinos are ultrarelativistic, this case will be realized with a very small probability.

2.3.2. The Second Case (the Case When Weak Interactions Do Not Generate Masses). In the framework of the electroweak model [15] the weak interactions are chiral-invariant and the masses of quarks, leptons, and gauge bosons are zero and the masses are generated by using Higgs mechanism [16]. So, the Lagrangian of standard weak interactions is chiral-invariant [15]; therefore, these interactions cannot generate masses. In that case it is perfectly warrantable to include all neutrino polarization energy W in the kinetic energy of the neutrino and to leave the neutrino mass without change ($p^2 \gg M^2$):

$$\sqrt{p^2+M^2}+W = \sqrt{p'^2+M^2} \rightarrow p'^2 = p^2+2W\sqrt{p^2+M^2}+W^2, \quad (36)$$

or

$$p' = \sqrt{p^2+2Wp+W^2+\frac{WM^2}{p}}.$$

If one neglects the forth term which is much smaller than the second term since $p^2 \gg M^2$, then the expression for p' gets the following simple form:

$$p' = p + W. \quad (37)$$

Wolfenstein's equation in this case looks like

$$i\frac{d\nu_{\rm ph}}{dt} = (\sqrt{p'^2 I + \hat{M}^2}\nu_{\rm ph}. \quad (38)$$

In the ultrarelativistic limit $\left(\sqrt{p'^2 I + \hat{M}^2} \simeq p'\hat{I} + \frac{\hat{M}^2}{2p'}\right)$, this evolution equation for the neutrino wave function ν_{ph} in the matter gets the following form:

$$i\frac{d\nu_{\mathrm{ph}}}{dt} = \left(p'\hat{I} + \frac{\hat{M}^2}{2p'}\right)\nu_{\mathrm{ph}}. \tag{39}$$

From Eq. (39) we obtain the same expression for a neutrino transition probability as for the case of neutrino vacuum oscillations where p changes to p':

$$P(E', t, \ldots) = 1 - \sin^2 2\theta \sin\frac{2\pi ct}{L'_0}, \tag{40}$$

where $E' \simeq p'c = pc + W$ and $L'_0 = \frac{4\pi E'_\nu \hbar}{\Delta m^2 c^3}$.

The same result was obtained when we worked in the framework of the scheme of masses mixings where we have taken into account that the standard weak interactions for their chiral invariance cannot generate masses [17, 18].

3. CONCLUSION

Wolfenstein's equation is used to describe the neutrino (particle) passing through the matter. Though this equation was obtained to describe the neutrino passing through the matter (by weak interactions which are left-side interactions), it is the Schrödinger type of equation and therefore this equation is the only one for left-right symmetric wave function and, correspondingly, it is valid for the left-right symmetric interactions.

It is supposed that while the neutrino is passing through the matter, the resonance enhancement of neutrino oscillations in the matter appears. It is shown that Wolfenstein's equation, describing passage of the neutrino through the matter, has a disadvantage (does not take into account the law of momentum conservation; i.e., it is supposed that in the matter the neutrino energy changes, but not its momentum). It leads, for example, to the change in the effective mass of the neutrino by the value $0.87 \cdot 10^{-2}$ eV from a very small value of the energy polarization of the matter caused by the neutrino, which is equal to $5 \cdot 10^{-12}$ eV. After removing this disadvantage (i.e., taking into account that neutrino momentum also changes in matter) we have obtained a solution to this equation. In this solution a very small enhancement of neutrino oscillations in the solar matter appears due to the smallness of the energy polarization of the matter caused by the neutrino.

Solutions to Wolfenstein's equation were also obtained for two limiting cases: when the full energy of neutrino interactions with matter is included in the neutrino mass and when the full energy of neutrino interaction with the matter is included in the neutrino kinetic energy.

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