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A. P. Severyukhin, V. V. Voronov, I. N. Borzov,
N. N. Arsenyev, Nguyen Van Giai *

COMPLEX CONFIGURATION EFFECTS ON β-DECAY RATES

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* Institut de Physique Nucléaire, CNRS-IN2P3 and University of Paris-Sud, Orsay, France

Северюхин А. П. и др. E4-2013-133
Влияние сложных конфигураций на периоды β-распада

На базе сепарабелизованного взаимодействия Скирма удалось одновременно учесть связь между одно- и двухфононными компонентами волновых функций, а также эффекты тензорного взаимодействия для описания гамов-теллеровских состояний. Показано, что влияние тензорных корреляций на фрагментацию гамов-теллеровских состояний приводит к заметному перераспределению силы переходов. Наряду с этим получено уменьшение периодов бета-распада. На примере нейтронно-избыточных изотонов с $N = 50$ продемонстрировано, что одновременный учет сложных конфигураций и тензорных корреляций приводит к хорошему описанию экспериментальных данных и позволяет предсказать период бета-распада ядра ^{76}Fe, что важно для астрофизических приложений.

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Complex Configuration Effects on β-Decay Rates

Starting from the separabelized Skyrme interaction we study the influence of the coupling between one- and two-phonon terms in the wave functions and the tensor force effects on properties of Gamow–Teller states. We observe a redistribution of the GT strengths due to the tensor correlation influence on the $2p$–$2h$ fragmentation of GT states. The β^--decay half-lives are decreased by these effects. As an example, we describe available experimental data for the $N = 50$ isotones and give predictions for ^{76}Fe which is important for stellar nucleosynthesis.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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1. INTRODUCTION

Many fundamental issues depend on our quantitative understanding of the β-decay phenomena in nuclei. Due to phase-space amplification effect, the beta rates are sensitive to both nuclear binding energies and the beta-strength function. Within appropriate beta-decay model, the correct amount of the integral beta strength, should be placed within the properly calculated Q_β-window provided that the spectral distribution is also close to the «true» beta-strength function. It is desirable to have theoretical models which can describe the data wherever they can be measured and which can predict the properties related to spin–isospin modes in systems too short-lived to be allowed for experimental studies. One of the successful tools for studying charge-exchange nuclear modes is the quasiparticle random phase approximation (QRPA) with the self-consistent mean field derived from a Skyrme-type energy-density functional (EDF), see, e. g., [1–4]. These QRPA calculations enable one to describe the properties of the ground state and excited charge-exchange states using the same EDF.

Experimental studies using the multipole decomposition analysis of the (*n, p*) and (*p, n*) reactions [5, 6] found substantial Gamow–Teller (GT) strength above the GT resonance peak and have clarified a longstanding problem of the missing experimental GT strength, hence resolved the discrepancy between the theoretical predictions within the RPA and the experimental measurements. It is necessary to take into account a coupling with more complex configurations that results in shifting some strength up [7–9]. Using the Skyrme EDF and the RPA, such attempts in the past [10, 11] have allowed one to understand the damping of charge-exchange resonances and their particle decay. Recently, the damping of the GT mode was investigated using the Skyrme-RPA plus particle-vibration coupling [12]. The main difficulty is that the complexity of the calculations increases rapidly with the size of the configuration space and one has to work within limited spaces.

Making use of the finite rank separable approximation (FRSA) [13, 14] for the residual interaction enables one to perform Skyrme-QRPA calculations in very large two-quasiparticle spaces. Taking into account the basic ideas of the quasiparticle–phonon model (QPM) [15], the approach has been generalized to take into account the coupling between one- and two-phonon components of the wave functions [16]. The FRSA has been used to study the electric low-lying states and giant resonances within the QRPA and beyond [14, 16, 17].

Recently, our approach has been proposed for the charge-exchange nuclear excitations [18]. The FRSA is then extended to accommodate tensor correlations to mimic the Skyrme-type tensor interactions [19]. In this paper, we generalize our approach to take into account the coupling between one- and two-phonon terms in the wave functions. As an application, we present the evolution of the β-decay scheme of the neutron-rich $N = 50$ isotones, in comparison to the doubly magic nucleus ^{78}Ni that is also an important waiting point in the r-process [20].

2. THE METHOD

2.1. FRSA Model. The starting point of the method is the HF-BCS calculation [21] of the parent ground state, where the spherical symmetry is imposed on the quasiparticle wave functions. In the particle–hole (p–h) channel we use the Skyrme interaction with the triplet-even and triplet-odd tensor interactions which were proposed in the pioneering works [22, 23]. The continuous part of the single-particle spectrum is discretized by diagonalizing the HF Hamiltonian on a harmonic oscillator basis. The inclusion of the tensor interaction results in the following modification of the spin-orbit potential in coordinate space [24, 25]:

$$U^{(q)}_{S.O.} = \frac{W_0}{2r}\left(2\frac{d\rho_q}{dr} + \frac{d\rho_{q'}}{dr}\right) + \left(\alpha\frac{J_q}{r} + \beta\frac{J_{q'}}{r}\right), \qquad (1)$$

where ρ_q and J_q ($q = n, p$) are the densities and the spin-orbit densities, respectively. The values α and β can be separated into contributions of the central force (α_c, β_c) and the tensor force (α_T, β_T) [24, 25].

We work in the quasiparticle representation defined by the canonical Bogoliubov transformation:

$$a^{+}_{jm} = u_j\alpha^{+}_{jm} + (-1)^{j-m}v_j\alpha_{j-m}, \qquad (2)$$

where α^{+}_{jm} (α_{jm}) is the quasiparticle creation (annihilation) operator and jm denote the quantum numbers $nljm$. The pairing correlations are generated by the density-dependent zero-range force

$$V_{\rm pair}(\mathbf{r}_1, \mathbf{r}_2) = V_0\left(1 - \eta\left(\frac{\rho\left(r_1\right)}{\rho_0}\right)^{\gamma}\right)\delta\left(\mathbf{r}_1 - \mathbf{r}_2\right), \qquad (3)$$

where the parameter ρ_0 is the nuclear saturation density. The values V_0, η and γ are fixed to reproduce the odd-even mass difference of corresponding nuclei [14, 17].

To build the QRPA equations on the basis of quasiparticle states as defined above and using consistently the residual interactions (derived from the Skyrme

EDF in the particle–hole channel and from the zero-range pairing force in the particle–particle channel) is a standard procedure [26]. This leads to the familiar QRPA equations in configuration space:

$$\begin{pmatrix} \mathcal{A} & \mathcal{B} \\ -\mathcal{B} & -\mathcal{A} \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = \omega \begin{pmatrix} X \\ Y \end{pmatrix}. \tag{4}$$

Solutions of this set of linear equations yield the eigen-energies ω and the amplitudes X, Y of the excited states.

The dimensions of the matrices $\mathcal{A}$ and $\mathcal{B}$ grow very rapidly with the size of the nuclear system unless severe and damaging cutoffs are made to the 2-quasiparticle configuration space. It is well known that, if the matrix elements of the $\mathcal{A}$ and $\mathcal{B}$ matrices take a separable form, the eigenvalues of Eq. (3) can be obtained as the roots of a relatively simple secular equation [15, 27]. In the case of the Skyrme interaction this feature has been exploited by different authors [13, 28, 29].

In particular, a method has been proposed in [13, 14] to calculate non charge-exchange excitations, and in [18] we have extended this method to the case of non closed-shell nuclei and charge-exchange excitations. The main step is to simplify the central particle–hole interaction V^{C}_{ph} by approximating it by its Landau–Migdal form. All Landau parameters with $l > 1$ are equal to zero in the case of Skyrme interactions. We keep only the $l = 0$ terms in V^{C}_{ph} and this approximation is reasonable [13, 14]. The two-body Coulomb residual interaction is dropped. Therefore we can write V^{C}_{ph} as

$$V^{a}_{\mathrm{res}}(\mathbf{r}_1, \mathbf{r}_2) = N_0^{-1} \left[F^{a}_0(r_1) + G^{a}_0(r_1)\sigma_1 \cdot \sigma_2 + \right. \\ \left. +(F^{'a}_0(r_1) + G^{'a}_0(r_1)\sigma_1 \cdot \sigma_2)\tau_1 \cdot \tau_2\right] \delta(\mathbf{r}_1 - \mathbf{r}_2), \tag{5}$$

where a is the channel index $a = \{ph, pp\}$; σ and $\tau^{(i)}$ are the spin and isospin operators, and $N_0 = 2k_F m^*/\pi^2\hbar^2$ with the Fermi momentum k_F and the nucleon effective mass m^*. $G^{pp}_0 = 0$, $G^{'pp}_0 = 0$; the expressions for $F^{ph}_0, F^{'ph}_0$, $G^{ph}_0, G^{'ph}_0$ and $F^{pp}_0, F^{'pp}_0$ can be found in [30] and [14], respectively. For the case of electric excitations one can neglect the spin–spin terms since they play a minor role. For the case of GT excitations we simplify the tensor $p{-}h$ interaction by replacing it by the two-term separable interaction as introduced in [31],

$$V^{ph}_{T}(\mathbf{r}_1, \mathbf{r}_2) = V_{T1}(\mathbf{r}_1, \mathbf{r}_2) + V_{T1}(\mathbf{r}_2, \mathbf{r}_1) + V_{T2}(\mathbf{r}_1, \mathbf{r}_2), \tag{6}$$

$$V_{T1} = \tau^{(1)}\tau^{(2)}\lambda_1 \sum_{M} T_{01M}(\hat{r}_1, \sigma_1) r_2^2 T^{*}_{21M}(\hat{r}_2, \sigma_2), \tag{7}$$

$$V_{T2} = \tau^{(1)}\tau^{(2)}\lambda_2 \sum_{M} r_1^2 T_{21M}(\hat{r}_1, \sigma_1) r_2^2 T^{*}_{21M}(\hat{r}_2, \sigma_2), \tag{8}$$

where the strengths λ_1 and λ_2 are adjusted to reproduce the centroid energies of the GT and spin-quadrupole strength distributions calculated with the original tensor $p{-}h$ interaction. The $p{-}h$ matrix elements and the antisymmetrized $p{-}p$ matrix elements can be written as the separable form in the angular coordinates [13, 14]. After integrating over the angular variables, the one-dimensional radial integrals are numerically calculated by choosing a large enough cutoff radius R and using an N-point integration Gauss formula with abscissas r_k and weights w_k [13]. Thus, one is led to deal with a problem where the matrix elements of $V_{\rm res}$ are sums of products and the number $\tilde{N}$ of terms in the sums depends only on N. In particular, $\tilde{N} = 4N + 4$ and $\tilde{N} = 6N$ are obtained for the cases of GT and electric excitations, respectively. One can call it a separable approximation of finite rank $\tilde{N}$, since finding the roots of the secular equation amounts to find the zeros of a $\tilde{N} \times \tilde{N}$ determinant, and the dimensions of the determinant are independent of the size of the configuration space, i. e., of the nucleus considered. The studies [14, 18] enable us to conclude that $N = 45$ is enough for the electric and charge-exchange excitations studied here in nuclei with $A \leqslant 208$.

2.2. Phonon–Phonon Coupling. In the next step, we construct the wave functions from a linear combination of one-phonon and two-phonon configurations

$$\Psi_\nu(JM) = \left(\sum_i R_i(J\nu) Q^+_{JMi} + \sum_{\lambda_1 i_1 \lambda_2 i_2} P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu) \left[Q^+_{\lambda_1 \mu_1 i_1} \bar{Q}^+_{\lambda_2 \mu_2 i_2} \right]_{JM} \right) |0\rangle, \quad (9)$$

$Q^+_{\lambda\mu i}|0\rangle$ ($\bar{Q}^+_{\lambda\mu i}|0\rangle$) is the GT (electric) excitation having energy $\omega_{\lambda i}$ ($\bar{\omega}_{\lambda i}$). The normalization condition for the wave functions (9) is

$$\sum_i R_i^2(J\nu) + \sum_{\lambda_1 i_1 \lambda_2 i_2} (P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu))^2 = 1. \quad (10)$$

The amplitudes $R_i(J\nu)$ and $P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu)$ are determined from the variational principle which leads to a set of linear equations

$$(\omega_{\lambda i} - \Omega_\nu) R_i(J\nu) + \sum_{\lambda_1 i_1 \lambda_2 i_2} U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji) P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu) = 0, \quad (11)$$

$$(\omega_{\lambda_1 i_1} + \bar{\omega}_{\lambda_2 i_2} - \Omega_\nu) P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu) + \sum_i U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji) R_i(J\nu) = 0. \quad (12)$$

The rank of the set of linear equations (11) and (12) is equal to the number of one- and two-phonon configurations included in the wave function (9).

Its solution requires to compute the matrix elements coupling one- and two-phonon configurations

$$U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji) = \langle 0|Q_{\lambda i} H \left[Q^{+}_{\lambda_1 i_1} \bar{Q}^{+}_{\lambda_2 i_2}\right]_J |0\rangle. \tag{13}$$

Equations (11) and (12) have the same form as the QPM equations [8,15], but the single-particle spectrum and the residual interaction are derived from the same Skyrme EDF.

3. RESULTS OF CALCULATIONS

We apply the approach to study the influence of the coupling between one- and two-phonon terms in the wave functions and the tensor force effects on the GT strength distributions of the neutron-rich $N = 50$ isotones. In particular, we focus on describing the β-decay half-lives since this integrated nuclear quantity is sensitive to the inclusion of the $[1^+_i \otimes \lambda^+_{i'}]$ terms, i. e., all electric phonons with $\lambda > 2$ vanish. Its experimentally known value puts an indirect constraint on the calculated GT strength distributions within the Q_β-window. To calculate the half-lives, the same ansatz as in Sec. II of [32] with the ratio of the weak axial-vector and vector coupling constants $G_A/G_V = 1.25$ [33] is used. In the allowed GT approximation, the β^--decay rate is expressed by summing the probabilities of the energetically allowed transitions (in units of $G_A^2/4\pi$) weighted with the integrated Fermi function

$$T^{-1}_{1/2} = \sum_k \lambda^k_{if} = D^{-1} \left(\frac{G_A}{G_V}\right)^2 \sum_k f_0(Z, A, E_i - E_{1^+_k}) B(GT)_k, \tag{14}$$

$$E_i - E_{1^+_k} \approx \Delta M_{n-H} + \mu_n - \mu_p - E_k, \tag{15}$$

where $D = 6147$ s [33], E_i denotes the ground-state energy of the parent nucleus, and $E_{1^+_k}$ is a state of the daughter nucleus (Z, A). $\Delta M_{n-H} = 0.782$ MeV is the mass difference between the neutron and the hydrogen atom, μ_n and μ_p are the neutron and proton chemical potentials, respectively. E_k and $B(GT)_k$ are the solutions either of the QRPA equations or of the set of linear equations (11) and (12) taking into account the two-phonon configurations. All calculations are performed without any quenching factor.

As the parameter set in the particle–hole channel, we use the central Skyrme interaction SGII [30] known to be successful describing the spin-dependent properties and the same zero-range tensor interaction as that in [31]. As proposed in [19], we assume the following parameters of the tensor $p{-}h$ interaction (6):

$$\lambda_1 = \frac{1300}{A^2}\,\mathrm{MeV}\cdot\mathrm{fm}^{-2}, \tag{16}$$

$$\lambda_2 = \frac{36}{A^2}\,\mathrm{MeV}\cdot\mathrm{fm}^{-4}. \tag{17}$$

These parameters mimic the original Skyrme tensor force with $\alpha_T = -180\ \mathrm{MeV}\cdot\mathrm{fm}^5$ and $\beta_T = 120\ \mathrm{MeV}\cdot\mathrm{fm}^5$, which appear in the spin-orbit potential (1). To get a reasonable description of the experimental pairing energies around the ^{90}Zr region, we employ a volume pairing force (3) with $\eta = 0$, acting in the particle–particle channel with a strength of $-270\ \mathrm{MeV}\cdot\mathrm{fm}^3$ and a pairing active space limited to an energy range of 10 MeV above the Fermi energies [17].

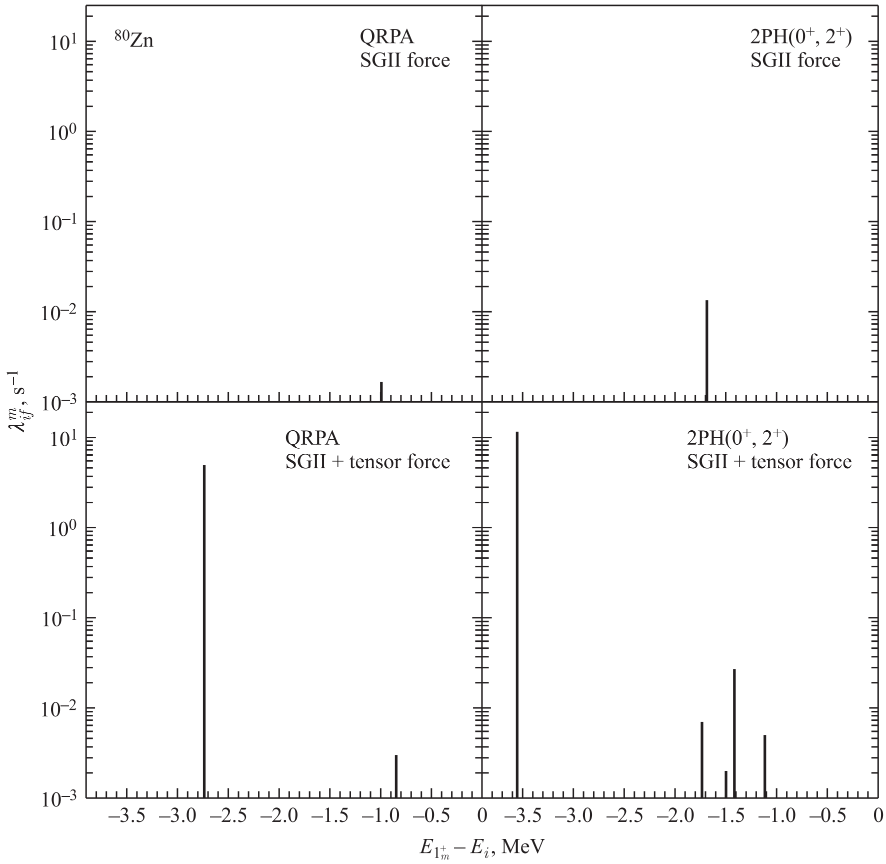

Fig. 1. The phonon–phonon coupling effect on probabilities of β transitions for ^{80}Zn. Results of the calculations without the tensor interaction and with the tensor interaction are shown

Taking into account both the tensor effect and the $[1_i^+ \otimes \lambda_{i'}^+]$ configurations, our results indicate a noticeable change of the GT strength distribution within the Q_β-window compared with those without the effects. For ^{80}Zn as an illustrative example, the probabilities of β transitions λ_{if} are shown in Fig. 1. One can see that there is the strong impact of the tensor force on the lowest peak. At the same time, the extension of the configuration space to the two-phonon term induces the 0.8 MeV downward shift. As a result, Fig. 2 shows the evolution of

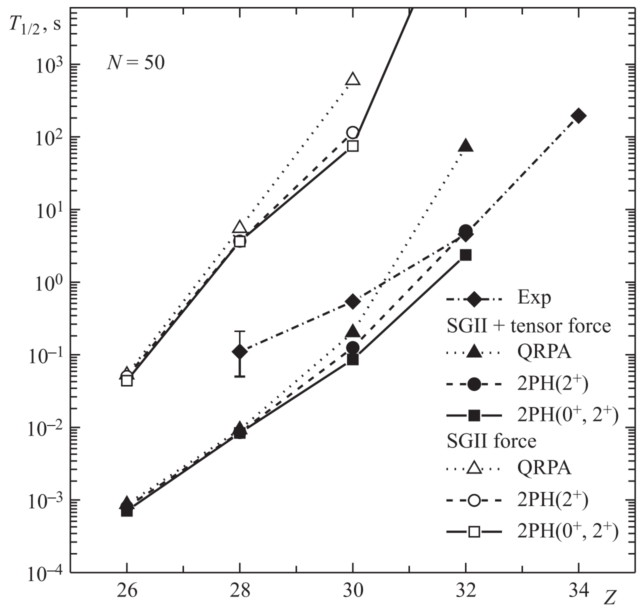

Fig. 2. The phonon–phonon coupling effect on β^--decay half-lives of the neutron-rich $N = 50$ isotones. Results of the calculations without the tensor interaction and with the tensor interaction are shown. Experimental data is taken from [34]

the β-decay half-lives of the neutron-rich $N = 50$ isotones. One can see that our results are in reasonable agreement with the experimental data [34]. It is shown that the inclusion of the tensor correlations leads to a reduction of β-decay half-life in the same way as was found in [4]. We provide this analysis taking into account the coupling between one- and two-phonon terms. We stress that the two-phonon term of the wave function (9) is dominated by the $[1_i^+ \otimes 2_{i'}^+]$ configuration (see Fig. 2). Since there is a clear influence of the 2_1^+ phonon on the half-lives,

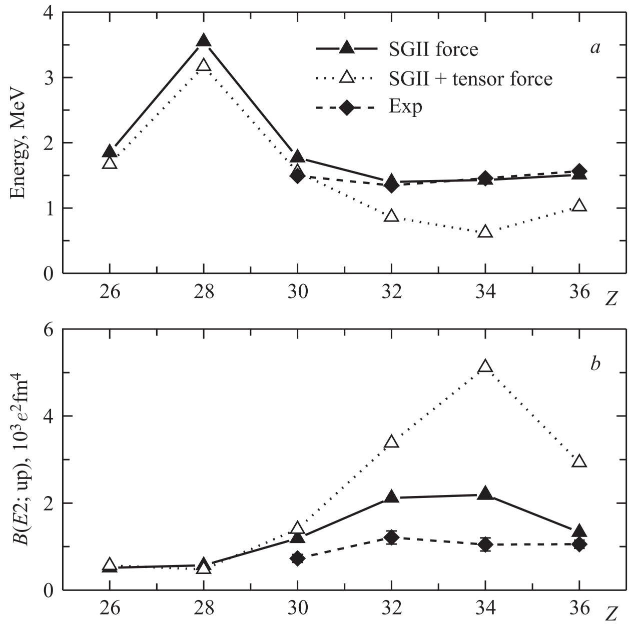

Fig. 3. Energies and $B(E2)$ values for up-transitions to the first 2^+ QRPA states in the neutron-rich $N = 50$ isotones. Experimental data are taken from [35]

we examine the energies and transition probabilities of the 2_1^+ QRPA states of the neutron-rich $N = 50$ isotones. As can be seen from Fig. 3, the FRSA model with the SGII+tensor interaction reproduces the experimental data [35] very well.

4. CONCLUSION

Starting from the Skyrme force the GT strength has been studied within the extended FRSA model including both the tensor interaction effect and $2p{-}2h$ configurations. The suggested approach enables one to perform the calculations in very large configurational spaces. It is shown that there is a redistribution of the GT strengths by the tensor correlation effects on the $2p{-}2h$ fragmentation of GT states. Taking into account of these effects results in a reduction of β-decay half-life. The coupling between one- and two-phonon terms containing the 2^+ phonon states is essential. Using the same set of parameters we describe available experimental data for the neutron-rich $N = 50$ isotones and give predictions for ^{76}Fe that is important for stellar nucleosynthesis.

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141980, г. Дубна, Московская обл., ул. Жолио-Кюри, 6.
E-mail: publish@jinr.ru
www.jinr.ru/publish/