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SELF-SIMILARITY OF LOW-p_T CUMULATIVE PION PRODUCTION IN PROTON–NUCLEUS COLLISIONS AT U70

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Самоподобие рождения кумулятивных пионов
с малыми p_T в протон-ядерных взаимодействиях на У-70

Данные по инклюзивным спектрам $\pi^{\pm}$-мезонов, образующихся в столкновениях $p + A$ на У-70 при импульсе налетающего протона $p_L = 15{-}65$ ГэВ/c и угле регистрации $\theta_{\rm lab} = 159°$, анализируются в рамках теории z-скейлинга. Проверяется гипотеза о самоподобии рождения кумулятивных пионов с малыми поперечными импульсами. Построена скейлинговая функция $\psi(z)$ для ядер Be, C, Al, Ti, Mo, W. Она выражается через инвариантное инклюзивное сечение пионов и среднюю плотность множественности заряженных адронов. Результаты анализа сравниваются с данными, полученными группами Дж. Кронина, Р. Суляева и Д. Джаффе, при $p_L = 70, 400$ ГэВ/c, $p_T > 1$ ГэВ/c и $\theta_{\rm cms} \simeq 90°$. Обсуждается микроскопический сценарий взаимодействия $p + A$ в рамках долей импульсов x_1, x_2. Получено указание на самоподобие рождения кумулятивных пионов с малыми p_T во взаимодействиях $p + A$. Универсальность формы функции $\psi(z)$ использована для предсказания спектров π^+- и π^--мезонов в столкновениях $p + A$ на ядрах Be, C, Al, Ti, Mo, W в глубококумулятивной области ($x_2 \gg 1/A$).

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Aparin A. A., Tokarev M. V. E2-2013-139
Self-Similarity of Low-p_T Cumulative Pion Production
in Proton–Nucleus Collisions at U70

The data on inclusive momentum spectra of $\pi^{\pm}$ mesons produced in $p + A$ collisions over the range $p_L = 15{-}65$ GeV$/c$ and $\theta_{\rm lab} = 159°$ at U70 are analyzed in the framework of z-scaling approach. Self-similarity of the pion production in the low-p_T cumualtive region is verified. Scaling function $\psi(z)$ for Be, C, Al, Ti, Mo, W nuclei is constructed. It is expressed via the invariant cross section and the average multiplicity density of charged particles. Results of the analysis are compared with data obtained by J. Cronin, R. Sulyaev, and D. Jaffe groups at $p_L = 70, 400$ GeV/c, $p_T > 1$ GeV/c and $\theta_{\rm cms} \simeq 90°$. A microscopic scenario of $p + A$ interactions in terms of momentum fractions x_1, x_2 is discussed. Indication of self-similarity of the low-p_T cumulative pion production in $p + A$ collisions was found. Universality of the shape of the scaling function is used to predict inclusive cross sections of π^+ and π^- pions produced in $p + A$ collisions on Be, C, Al, Ti, Mo, W targets in the deep-cumulative region ($x_2 \gg 1/A$).

The investigation has been performed at the Veksler and Baldin Laboratory of High Energy Physics, JINR.

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INTRODUCTION

The heavy ion experimental programs at the Relativistic Heavy Ion Collider at BNL [1, 2], Super Proton Synchrotron [3], and Large Hadron Collider at CERN [4–8] are aimed to study nuclear matter under extreme conditions (high collision energy, high energy density and high multiplicity). It is expected that in heavy-ion collisions the collective phenomena should play an important role. The special interest is related with searching for signatures of the phase transitions and critical points. The hypothesis of self-similarity is an important concept to study particle interactions at high energies [9–19].

The general concepts in the critical phenomena are related with the notions of "scaling" and "universality" [20]. Scaling means that the system near the critical point exhibiting self-similar properties is invariant under transformation of the scale. According to universality, quite different systems behave in a remarkably similar way near the respective critical point. It is assumed that transition of the nuclear matter from the hadron to quark and gluon degrees of freedom near the critical point should reveal large fluctuations, correlations and discontinuty of some experimental quantities characterizing the system.

Extreme conditions of particle production can be created by compressing nuclei in the cumulative processes. Such processes have been extensively studied at JINR, ITEP, and IHEP (see [13–15, 21, 22] and references therein). They can be investigated at RHIC and LHC in a new kinematic region [23]. Search for new phenomena in the cumulative processes at future acceleraters FAIR (GSI) and NICA (JINR) has been suggested in [24, 25]. The anisotropy of momentum space at small scales in the deep-cumulative processes has been discussed in the framework of z-scaling concept.

We briefly remind here that the concept of z-scaling [26, 27] is based on the principles of self-similarity, locality, scale relativity and fractality of constituent interactions at small scales. A-dependence of z-scaling in $p + A$ collisions at high transverse momentum p_T of inclusive particle and scattering angle $\theta_{\rm cms} \simeq 90^\circ$, was studied in [28]. Universality of the shape of scaling function $\psi(z)$ for different nuclei was found. Self-similarity of the low-p_T cumulative pion production in $p + A$ collisions at energy $\sqrt{s} \simeq 27.4$ GeV and angle

$\theta_{\rm lab} = 70°-160°$ at FNAL [29] has been analyzed in [30]. New indication of universality of $\psi(z)$ in the noncumulative high-p_T [31] and the cumulative low-p_T [29] regions was found. This conclusion is supported by analysis results [32] of the data [33] on momentum spectra covering the cumulative high-p_T region for $h^{\pm}$ hadrons produced in $p + A$ collisions at $p_L = 50$ GeV/c and angle $\theta_{\rm lab} = 35°$ at U70.

In the present work, low-p_T data [22] of the cumulative pion production in $p + A$ collsions obtained at the U70 accelerator complex are analyzed. Self-similarity of the pion production in this kinematic region is verified. The paper is organized as follows. The main features of z-scaling as the method of data analysis for hadron–nucleus collisions are briefly described in Sec. 1. Inclusive spectra of cumulative pions produced in $p + A$ collisions obtained at $p_L = 17-58$ GeV/c and $\theta_{\rm lab} = 159°$ are discussed in Sec. 2. Results of data analysis in the framework of the z-scaling approach are given in Sec. 3. Comparison of the results with the data obtained by J. Cronin, R. Sulyaev, G. Leksin and D. Jaffe groups at higher energies is presented. Universality of the shape of the scaling function is discussed in Sec. 4. Predictions of the pion spectra in $p + A$ collisions in the deep-cumulative region are given in Sec. 5. In Sec. 6, a microscopic scenario of the elementary subprocess in terms of momentum fractions x_1 and x_2 is presented. The obtained results are discussed in Sec. 7 and briefly formulated in Conclusions.

1. z-SCALING

Self-similarity is a scale-invariant property related with dropping of certain dimensional quantities out of the physical picture of constituent interactions. It means that dimensionless quantities to describe the physical processes are used. In our case we construct the dimensionless scaling function $\psi(z)$ and self-similarity parameter z which are expressed via the measurable quantities such as inclusive and total inelastic cross sections, a multiplicity density, momenta and masses of colliding and produced particles.

In this section we follow the approach suggested in [28] which has been used for data analysis in [30,32]. It is assumed following Stavinsky's idea [14] that the gross feature of the inclusive particle distribution of the process $P_1 + P_2 \to p + X$ at high energies can be described in terms of the corresponding kinematic characteristics of the constituent subprocess. The subprocess satisfies the following condition [14]:

$$(x_1P_1 + x_2P_2 - p)^2 = (x_1M_1 + x_2M_2 + m_2)^2. \quad (1)$$

The equation is the expression of locality of the hadron interaction at a constituent level. The x_1 and x_2 are the fractions of incoming 4-momenta P_1 and P_2 of the colliding objects with masses M_1 and M_2. They determine the minimum energy necessary to produce a secondary particle with mass m_1 and four-momentum p. The parameter m_2 is introduced to satisfy the internal conservation laws (for electric and baryon charges, isospin, strangeness, and so on). Equation (1) reflects the minimum recoil mass hypothesis in the elementary subprocess. The quantity Ω is introduced to connect kinematic and structural characteristics of the interaction. It is chosen in the following form:

$$\Omega(x_1, x_2) = (1 - x_1)^{\delta_1}(1 - x_2)^{\delta_2}. \quad (2)$$

The parameters δ_1 and δ_2 are the fractal dimensions of the colliding objects. The fractions x_1 and x_2 are determined [28] to maximize the value of $\Omega(x_1, x_2)$, simultaneously fulfilling condition (1):

$$d\Omega(x_1, x_2)/dx_1|_{x_2=x_2(x_1)} = 0. \quad (3)$$

The fractions change over the range $0 < x_{1,2} < 1$ and cover the full phase space which is accessible at any collision energy.

The scaling function is written in the following form:

$$\psi(z) = -\frac{\pi s}{(dN/d\eta)\sigma_{\rm in}} J^{-1} E\frac{d^3\sigma}{dp^3}. \quad (4)$$

Here, $Ed^3\sigma/dp^3$ is the invariant cross section, $dN/d\eta$ is the multiplicity density as a function of the center-of-mass collision energy squared s and pseudorapidity η, $\sigma_{\rm in}$ is the inelastic cross section, J is the corresponding Jacobian [28].

The function $\psi(z)$ is normalized as follows:

$$\int_0^\infty \psi(z)dz = 1. \quad (5)$$

This relation allows us to interpret the function as a probability density to produce a particle with the corresponding value of variable z.

Self-similarity of an object revealing itself over a wide scale range is the general property of fractality. It means that the measure corresponding to the object diverges in terms of the resolution. In our case this measure is the self-similarity parameter z which has the following form:

$$z = z_0\Omega^{-1}. \quad (6)$$

Here, $z_0 = \sqrt{\hat{s}_\perp}/[m(dN_{\rm ch}/d\eta)]$ is the finite part of z. It is expressed via the ratio of the transverse energy $\sqrt{\hat{s}_\perp}$ released in the binary collision of constituents and the average multiplicity density of charged particles $dN_{\rm ch}/d\eta$ at $\eta = 0$ and the nucleon mass m. The divergent part Ω^{-1} describes the resolution at which the collision of the constituents can be singled out of this process. The $\Omega(x_1, x_2)$ represents a relative number of all initial configurations containing the constituents which carry fractions x_1 and x_2 of the incoming momenta. The δ_1 and δ_2 are fractal dimensions of the colliding objects. The momentum fractions x_1 and x_2 are determined to minimize the resolution $\Omega^{-1}(x_1, x_2)$ of measure z with respect to all possible subprocesses under condition (1).

Note that ψ and z are scale-dependent quantities. They both depend on the dimensional variables $\sqrt{s}$ and p_T. We assume that the hadron and nucleus interactions at high energies and high transverse momenta are interactions of fractals. In this region the internal structure of hadrons, interactions of their constituents and mechanism of hadronization should reveal self-similarity.

2. LOW-p_T CUMULATIVE PION PRODUCTION IN $p + A$ COLLISIONS AT U70

Cumulative particles are the particles produced in the kinematic region forbidden for free nucleon–nucleon interactions [13–15] (see also [21]). Such particles can be produced only in the processes with the participation of nuclei. Cumulative processes have been traditionally studied at small p_T. It corresponds to particle production in the backward semisphere (the large particle produced angle in the laboratory system frame). Another possibility to study these processes is an investigation of the cumulative particle production at high p_T [16]. The study of the cumulative processes is of great interest to search for signatures of the phase transition in highly compressed nuclear matter.

The low-p_T invariant inclusive spectra of $\pi^{\pm}$-meson production in $p + A$ collisions at a laboratory angle of 159° for nuclear targets: Be, C, Al, Ti, Mo, W have been given in [22]. The energy range of projectile protons varied from 15 to 65 GeV. The momentum range $p > 0.5$ GeV/c corresponds to the cumulative pion production. The data have been taken with an internal target of the U70 proton synchrotron (IHEP, Serpukhov).

Figure 1 shows the inclusive cross sections for π^+ (*a, c*) and π^- (*b, d*) pions produced in $p + A$ collisions for low-momentum $p_L = 15-20$ GeV/c and high-momentum $p_L = 53-64$ GeV/c intervals and angle $\theta_{\rm lab}$ of 159°. All further calculations have been performed at mean values of momentum p_L of 18 and 58 GeV/c, respectively.

As noted in [22], pion spectra for all nuclei demonstrate a similarity of form, and they are well approximated by $\exp(-T/T_0)$ at $T > 0.35$ GeV (T is the kinetic energy of the pion) just above the kinematic limit for pion production on quasi-free nucleons of the nucleus. The parameter T_0 ("temperature" of the

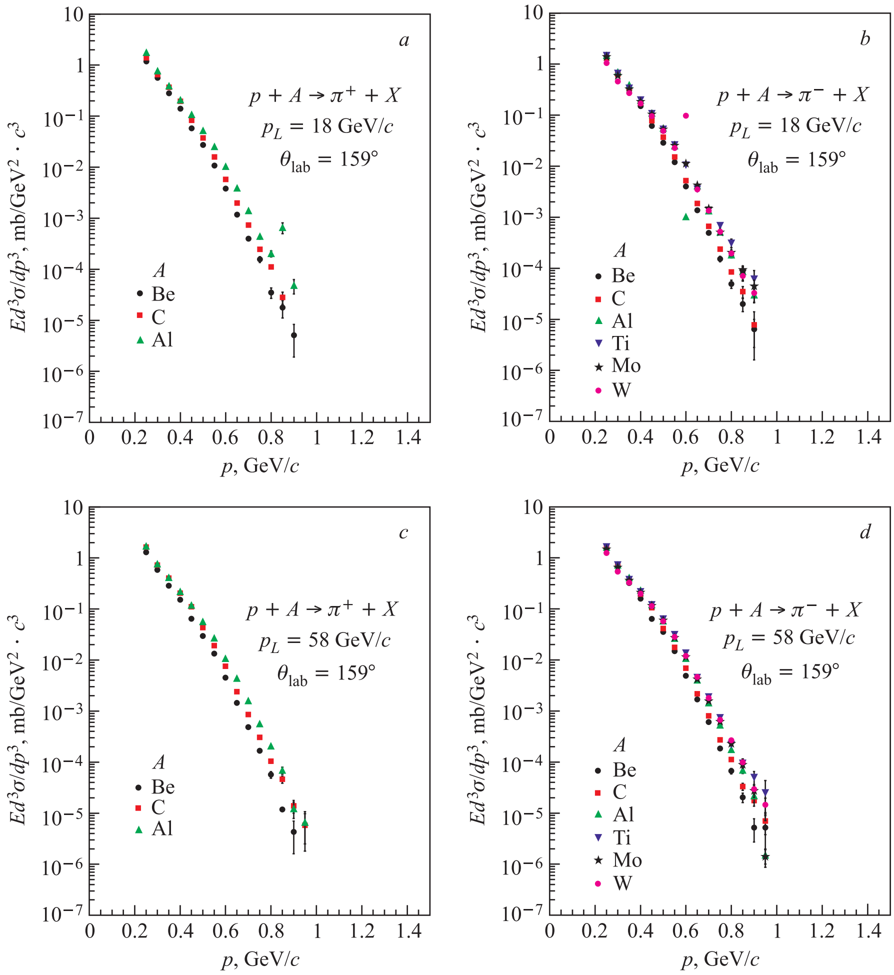

Fig. 1. The dependence of cross sections of π^+ (*a, c*) and π^- (*b, d*) pions produced in $p + A$ collisions on momentum p at $p_L = 18, 58$ GeV/c and angle $\theta_{\rm lab} = 159°$. The experimental data are taken from [22]

T-spectrum) was found to be weakly dependent on the atomic number of the target nucleus. This value increases from 45 to 54 MeV with the change of atomic weight A from 9 to 184. As seen from Fig. 1, the spectra decrease by 5 orders of magnitude in the range $p = 0.3$–0.9 GeV/c. The values of the transverse momentum p_T do not exceed 0.35 GeV/c.

3. INCLUSIVE PION SPECTRA IN z PRESENTATION

In the present paper we follow the procedure of data analysis proposed in [28] and used in [30,32]. The function ψ is calculated for every nucleus using Eq. (4) with the normalization factor $\sigma_{\rm in}^{pA}/\sigma_{\rm in}^{pp}$ instead of the total inelastic cross section for $p+A$ interactions $\sigma_{\rm in}^{pA}$. The multiplicity density of charged particles for $p+A$ collisions is parametrized by the following formula:

$$\rho(s, A) \simeq 0.67 \cdot A^{0.18} \cdot s^{0.105}, \quad A \geqslant 2. \tag{7}$$

Figure 2 shows z presentation of the data [22] for π^+ (a) and π^- (b) mesons. One can see that the the same curve describes $\psi(z)$ for Be, C, Al, Ti, Mo and W nuclei. It was found that the curve for π^- mesons coincides with similar one for D nucleus at $p_L = 70, 400$ GeV/c and $\theta_{\rm cms} \simeq 90^\circ$ [31, 34, 35]. For π^+ mesons the shape of ψ is practically the same. The values of the normalization factor are found to be different. It is related with the increase of π^+/π^- yield ratio as a collision energy decreases.

The symmetry transformation

$$z \to \alpha(A) \cdot z, \quad \psi \to \alpha^{-1}(A) \cdot \psi \tag{8}$$

of the scaling function and the self-similarity parameter were used to compare functions ψ for different nuclei. The A-dependence of the parameter α is described by the expression $\alpha(A) = 0.9A^{0.15}$ [28]. We verify the hypothesis of universality of the shape of ψ. This hypothesis means the validity of self-similarity of the hadron production in $p + A$ collisions over a wide kinematic region. The curve obtained for D nucleus at $p_L = 70, 400$ GeV/c and $\theta_{\rm cms} \simeq 90^\circ$ [31, 34, 35] is used as a reference curve. In the general case the multiplicity density $\rho(s, \eta, A)$ depends on the angle of produced particle. The pseudorapidity $\eta = -\ln({\rm tg}(\theta_{\rm cms}/2))$ is expressed via the angle $\theta_{\rm cms}$. The normalization condition (5) takes into account the angular dependence of $dN/d\eta$. We have no experimental information on this dependence. Therefore, we parameterize the function $\rho(s, \eta, A)$ in the form $\rho(s, \eta, A) = \rho(s, A) \cdot \chi(\theta_{\rm lab}, A)$ and fix the value of $\chi(\theta_{\rm lab}, A)$ for each nucleus so as to restore the shape of the reference curve. Results of this analysis are presented in Table 1. We found that the values of χ decrease with atomic weight A. Note that normalization of $\psi(z)$ at one point allows us to describe the function over a wide range of z. This result confirms the hypophesis of universality of the shape of the scaling function. Nevertheless, the direct measurements of the angular dependence of the multiplicity density of pions at this energy are necessary to verify this assumption.

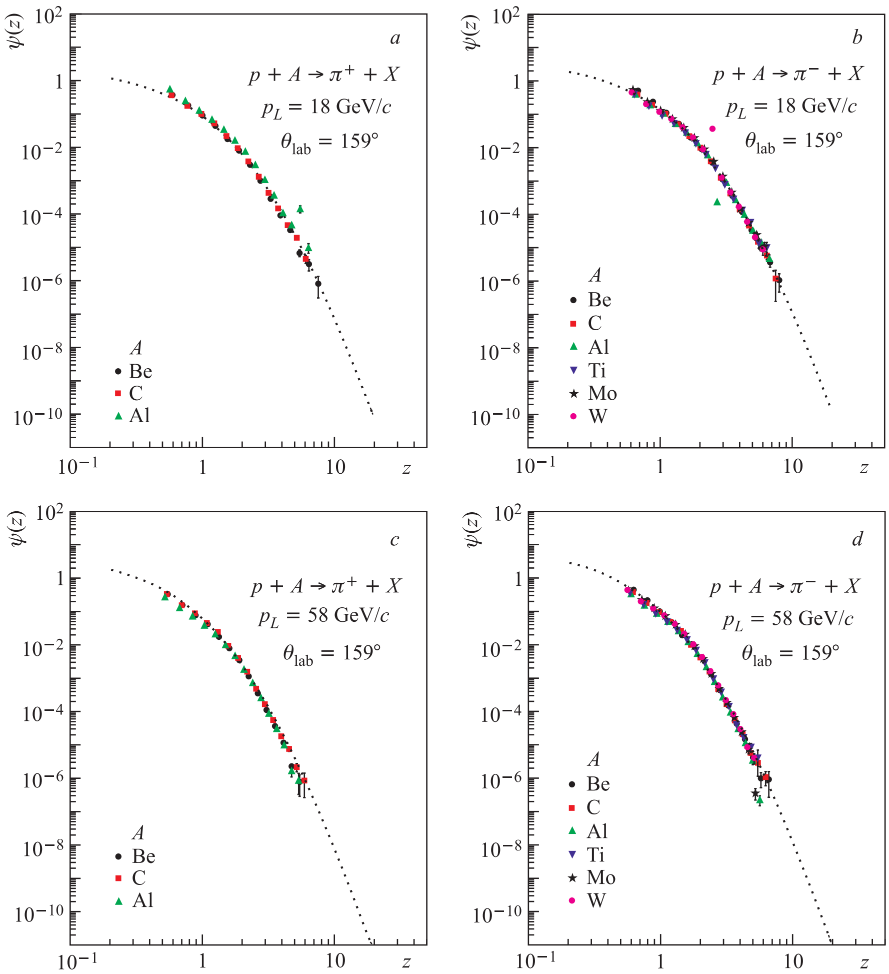

Fig. 2. Scaling function $\psi(z)$ versus variable z for π^+ (*a, c*) and π^- (*b, d*) pions produced in $p + A$ collisions at $p_L = 18, 58$ GeV/*c* and angle $\theta_{\rm lab} = 159^\circ$. The experimental data shown by symbols are taken from [22]. The dotted lines are the results of fitting the experimental data on inclusive momentum spectra taken from [31, 33–35]

The reference curve shown by the dotted line in Fig. 2 covers the range $z = 0.2-20$. We would like to note that the data points [22] cover both the preasymptotic ($z < 4$) and asymptotic ($z > 4$) regions. The latter is characterized by the power behavior of $\psi(z)$. The shape of $\psi(z)$ found in the present analysis

Table 1. The angular dependence of $\chi(\theta_{\rm lab}, A)$ for $\pi^{\pm}$ mesons produced in $p + A$ collisions at $p_L = 18$ and 58 GeV/c and $\theta_{\rm lab} = 159^{\circ}$

A	Be	C	Al	Ti	Mo	W
$p_L = 18$ GeV/c, $\theta_{\rm lab} = 159^{\circ}$ $\chi(\theta_{\rm lab}, A)$						
π^-	1.0	0.98	0.72	0.60	0.35	0.30
π^+	1.0	0.98	0.70			
$p_L = 58$ GeV/c, $\theta_{\rm lab} = 159^{\circ}$ $\chi(\theta_{\rm lab}, A)$						
π^-	1.0	0.97	0.65	0.44	0.20	0.15
π^+	1.0	0.97	0.55			

coincides with the shape of the scaling function found in [31, 34, 35] for the noncumulative region with high p_T. Values of the fractal dimensions for nucleon δ_N and nucleus δ_A for both regions are used to be the same: $\delta_N = \delta = 0.5$ and $\delta_A = A\delta$. It was assumed in [24, 25] that these dimensions should change in the cumulative region. The discontinuty of fractal dimension δ_A at high z was proposed to be the signature of the phase transition. Measurements of cross sections in the cumulative region for high z allow us to verify the power law, $\psi(z) \sim z^{-\beta}$, and the additive law, $\delta_A = A\delta$.

4. THE SHAPE OF $\psi(z)$

One of the features of z-scaling is the universality of the shape of the function $\psi(z)$. It has been found [28,30,32] that this property is valid for charged hadrons produced in $p + A$ collisions over a wide range of momenta and angles of the inclusive particle [29, 31, 34, 35]. Moreover, the flavor independence of $\psi(z)$ in $p + p$ collisions has been established in [27]. This feature is interpreted as self-similarity of the hadronization process. It is usually assumed that the constituent interactions and the hadronization process are modified by the nuclear medium. This effect is clearly manifested in high-energy collisions of heavy ions [1, 2]. Therefore, a flavor content as the property of each hadron can be considered as a sensitive probe of the nuclear matter state. It is assumed that the nuclear matter produced in the cumulative region is more compressed than in the usual state. It can be compressed considerably higher in the deep-cumulative region. Therefore, verification of the universality of the shape of $\psi(z)$ and the additive law for fractal dimension δ_A in this region is of particular interest.

We use the parameterization of the scaling function in the Tsallis form [36]:

$$\psi(z) = C[1 + (q-1)z/T]^{1/(1-q)}. \quad (9)$$

This form is flexible enough to describe $\psi(z)$ in noncumulative (see [37] and references therein) and cumulative [30, 32] regions. As seen from (9), the fitting parameters, C, q and T, are dimensionless. The first one is responsible for the absolute normalization of ψ; the second, for the asymptotics of ψ. The third parameter describes the transition regime of ψ from low to high z. The asymptotics of ψ are as follows: $\psi(z) \to C$, if $z \to 0$, and $\psi(z) \to z^{1/(1-q)}$, if $z \to \infty$.

Table 2. The fitting parameters of the function $\psi(z)$ for $\pi^{\pm}$ mesons produced in $p + A$ collisions at $p_L = 18, 58$ GeV/c and $\theta_{\text{lab}} = 159°$

Meson	π^+		π^-	
p_L, GeV/c	18	58	18	58
q	1.083	1.083	1.083	1.083
T	0.257	0.184	0.257	0.184
C	2.5	5.0	4.0	8.0

In the present analysis we have used the high-p_T data for D nucleus [31, 34, 35] to determine the fitting parameters. The values of the parameters C, q, T for π^+ and π^- mesons at $p_L = 18, 58$ GeV/c are presented in Table 2. The values of the parameter q for π^- and π^+ mesons were taken to be equal to $q = 1.083$. This value is defined by the asymptotics of $\psi(z)$ for high-p_T data [31, 34, 35]. The values of the parameter T are found to be different for low- and high-momentum intervals. The dotted lines shown in Fig. 2 are our calculations of $\psi(z)$ in the cumulative region.

5. LOW-p_T SPECTRA IN THE CUMULATIVE REGION

The universal shape of the scaling function allows us to predict inclusive spectra in the region which has not been available up to now. It is expected that the mechanisms of the hadron production in the cumulative and noncumulative regions differ from each other. As a result, the additive law, $\delta_A = A\delta$, for fractal dimensions of nuclei could be violated. The results obtained in Sec. 3 have shown that the shape of $\psi(z)$ is the same over a wide z region. Therefore, the experimental study of the hadron production in the cumulative region for $p > 1.5$ GeV/c ($z > 20$) is highly desirable to test this property.

The main physical motivation to investigate the cumulative processes is related with the assumption that the cumulative region corresponds to the extremal regime of particle production in which the nuclear matter is strongly compressed. We also assume that a nucleus size in the deep-cumulative region (the region near the kinematic boundary of the reaction) could be of the order of a nucleon

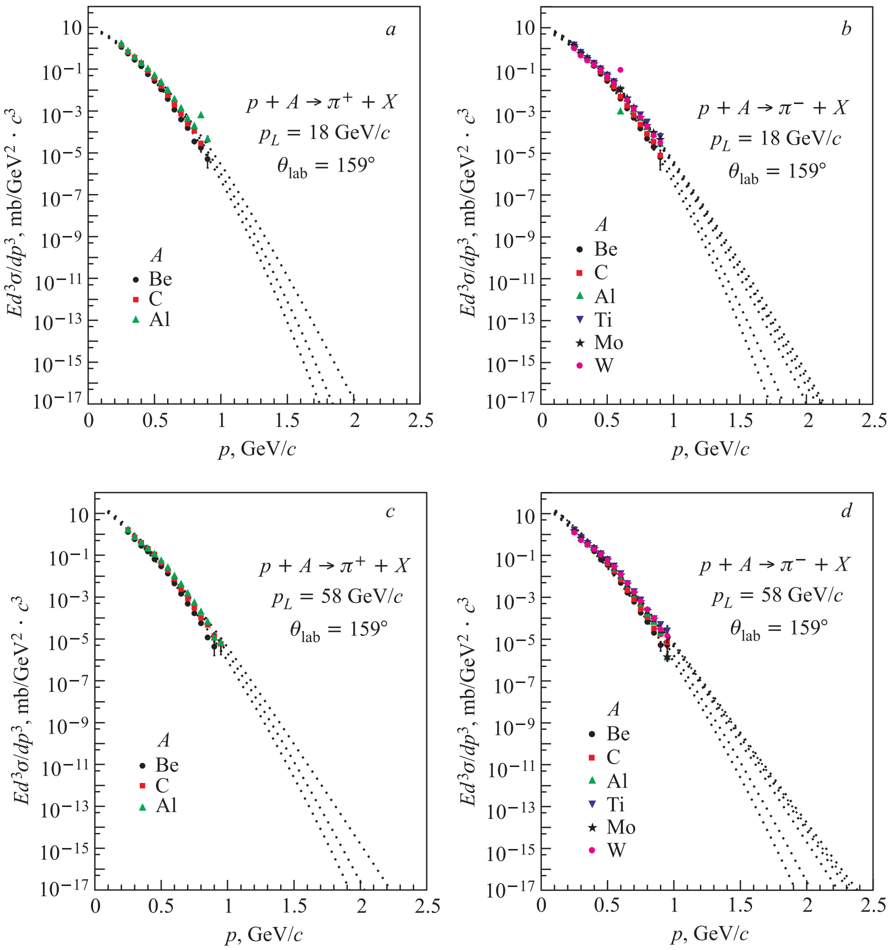

Fig. 3. The dependence of the cross sections of π^+ (*a, c*) and π^- (*b, d*) pions produced in $p + A$ collisions on momentum p at $p_L = 18, 58$ GeV/c and angle $\theta_{\rm lab} = 159^\circ$. The dotted lines are predictions based on z-scaling. The experimental data taken from [22] are shown by symbols

size. For this process the momentum of the inclusive particle should be fully balanced by the momentum of the recoil compressed system consisting of very slow constituents. The system in this state should demonstrate collective properties. Therefore, a transition regime from the single to multiple constituent interactions is expected.

Our predictions are based on self-similarity of the constituent interactions in the noncumulative region at high p_T. Therefore, extrapolation of the cross section far from the nucleon–nucleon kinematical boundary could allow us to verify simultaneously the power law, $\psi(z) \sim z^{-\beta}$, and search for its violation. Verification of self-similarity of hadron production in the cimulative region and low-p_T could give us a new insight into collective phenomena of the nuclear matter.

Figure 3 shows the dependence of inclusive cross section $Ed^3\sigma/dp^3$ on momentum p of π^+ (*a, c*) and π^- (*b, d*) mesons produced in $p + A$ collisions at $p_L = 18, 58$ GeV/*c*) and angle $\theta_{\rm lab} = 159°$. The symbols are the experimental data taken from [22]. Our predictions of spectra based on z-scaling are shown by the dotted lines. As seen from Fig. 3, the cross sections decrease with the momentum by more than seventeen orders of magnitude at $p = 2.$ GeV/*c*.

Table 3. The kinematic boundary for π^- mesons produced in $p + A$ collisions at $p_L = 18, 58$ GeV$/c$ and $\theta_{\rm lab} = 159°$

A	Proton	Be	C	Al	Ti	Mo	W
$p_L = 18$ GeV$/c$, $\theta_{\rm lab} = 159°$							
$p_{\rm max}$, GeV$/c$	0.431	3.30	4.14	7.14	9.56	12.2	14.1
$p_L = 58$ GeV$/c$, $\theta_{\rm lab} = 159°$							
$p_{\rm max}$, GeV$/c$	0.462	3.98	5.19	10.5	16.3	25.4	34.5

Kinematic boundaries for π^--meson production in $p + A$ collisons for low-momentum and high-momentum energy intervals and angle $\theta_{\rm lab} = 159°$ are shown in Table 3. Experimental measurements of the spectra over the range $p = 1{-}2$ GeV/*c* at $\theta_{\rm lab} = 159°$ (the deep-cumulative region) would allow us to test the power law of $\psi(z)$ in the interval $z = 10{-}10^2$.

6. MOMENTUM FRACTIONS x_1, x_2 AND $z{-}p$ PLOT

The use of Eqs. (1)–(3) allows us to analyze the kinematics of constituent subprocess in terms of the momentum fractions x_1 and x_2. The dependence

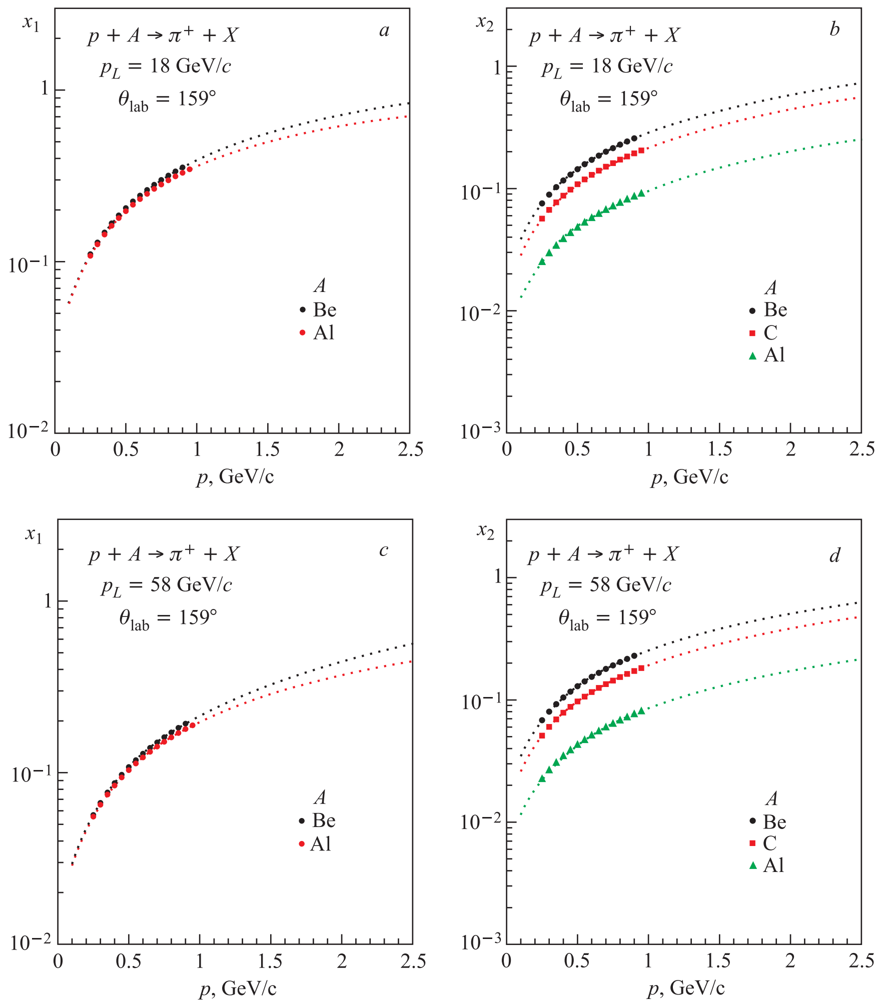

Fig. 4. The dependence of the fractions x_1 and x_2 on the momentum p of the π^+ mesons produced in $p + A$ collisions at $p_L = 18, 58$ GeV/c and $\theta_{\rm lab} = 159^\circ$

of these fractions on the momentum p for π^+ and π^- mesons produced in $p + A$ collisions at $p_L = 18, 58$ GeV/c and angle $\theta_{\rm lab} = 159^\circ$ are shown in Figs. 4 and 5, respectively. The fragmentation of the incident proton and nucleus are described by the fractions x_1 and x_2, respectively. Both ones are normalized as follows: $0 < x_1, x_2 < 1$. The fractions increase with the momentum p.

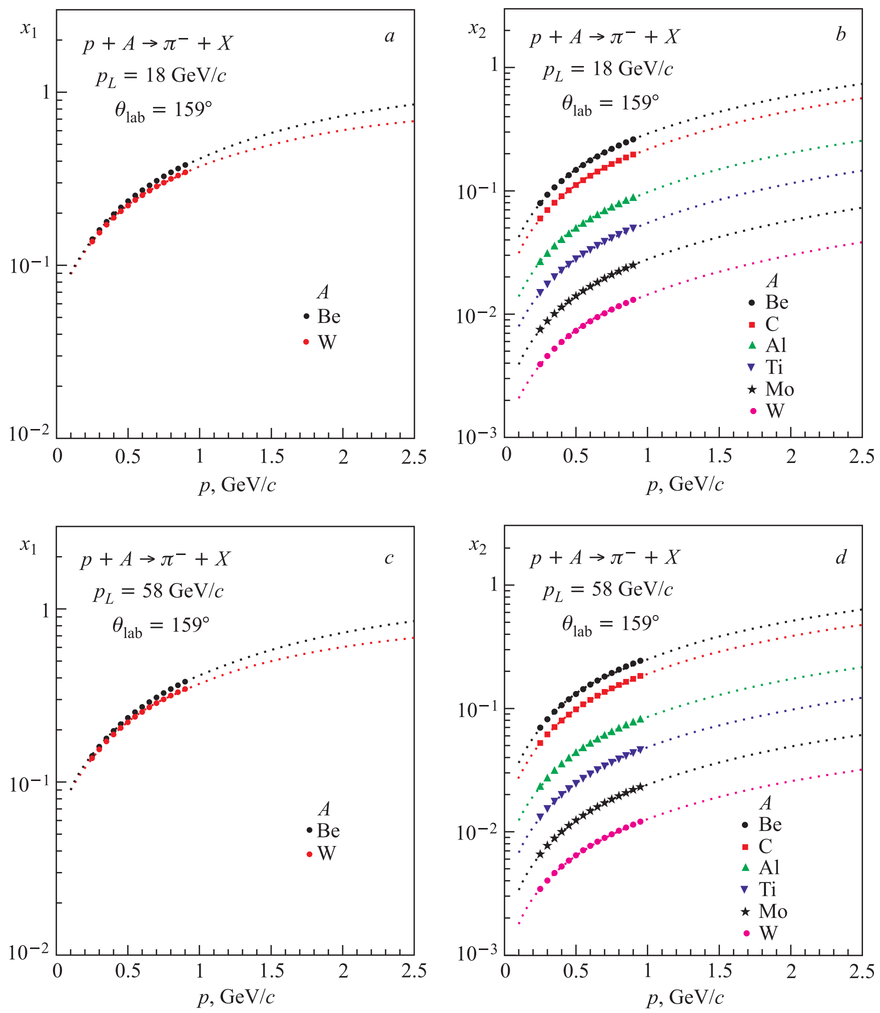

Fig. 5. The dependence of the fractions x_1 and x_2 on the momentum p of π^- mesons produced in $p + A$ collisions at $p_L = 18, 58$ GeV/c and $\theta_{\rm lab} = 159°$

The noncumulative region covers the range $x_2 < 1/A$ and the cumulative one $x_2 > 1/A$ for fragmentation of a nucleus. The cumulative region in this kinematics ($p_L = 18, 58$ GeV/c and $\theta_{\rm lab} = 159°$) corresponds to $p > 0.5$ GeV/c.

The z–p plot (Fig. 6) allows us to choose the most suitable kinematic region to search for new physics phenomena in $p + A$ collisions. The available

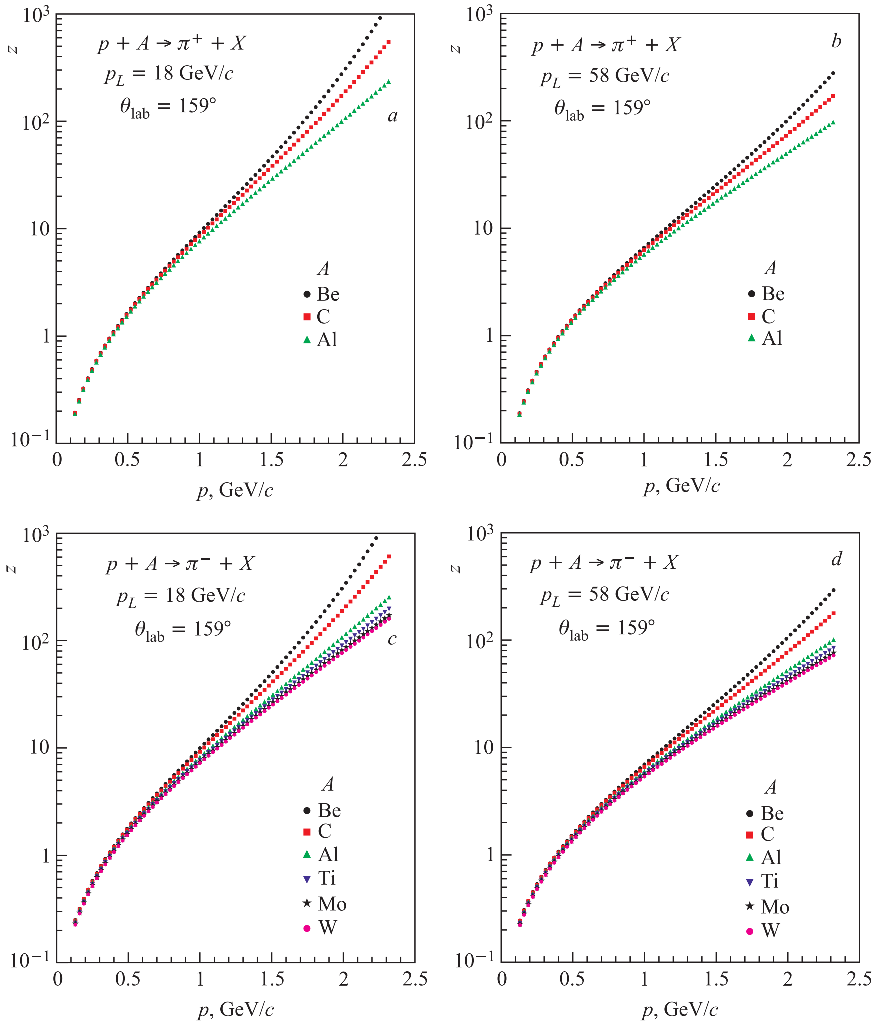

Fig. 6. The z–p plot for π^+ (*a, b*) and π^- (*c, d*) pions produced in p+A collisions at $p_L = 18, 58$ GeV/c and $\theta_{\rm lab} = 159°$

experimental high-p_T data cover the range of the self-similar parameter z up to 20. A new regime of hadron production is expected in the deep-cumulative region. The results shown in Fig. 2 demonstrate the transition to the power behavior of $\psi(z)$. Therefore, verification of the power law is of interest.

7. DISCUSSION

Self-similarity of constituent interactions at high energies has been studied in the z-scaling approach for different inclusive processes in pp and $\bar{p}p$ collisions (see [27] and references therein). The flavor independence of the scaling function over a wide range of z has been found. This result cogently indicates fractal properties of the internal hadron structure, constituent interactions and hadronization process. A more sophisticated approach developed in [26] allowed us to analyze the hadron spectra in $A + A$ collisions as well [38–40]. The main goal of the study is to search for signatures of the phase transitions of the nuclear matter and location of the critical point. A constituent energy loss as a function of energy and centrality of the collision and transverse momentum of the inclusive particle is assumed to be an important feature of the produced medium. The energy loss was found to increase with energy $\sqrt{s}$ and multiplicity density and decrease while the momentum p_T increases.

Complementary information on constituent interactions and properties of multiparticle systems can be obtained in $p + A$ collisions. We expect that such information would allow us to clarify properties of the transition from hadron to nuclear medium at different scales. Modification of the elementary subprocess is assumed to be stronger in the region forbidden for the particle production on free nucleon. This region is known as a cumulative one. The high-p_T and low-p_T cumulative pion production can be studied under the same (z-scaling) approach. Comparison of results of analysis of the above-mentioned and noncumulative high-p_T processes could give new indications of validity of self-similarity of the pion production over a wide scale range.

The experiment [22] performed by the Zolin group at U70 has measured the spectra of pions produced in $p + A$ collisions in the cumulative low-p_T region over the range $p_L = 15{-}65$. Similar measurements have been carried out by Leksin group [15] at $p_L = 400$ GeV/c at FNAL. The maximum value of the momentum p was reached up to 1 GeV/c. The decrease of the cross section is larger than five orders of magnitude. The function $\psi(z)$ (see Fig. 2) demonstrates transition to the power behavior for $z > 4$. The shape of $\psi(z)$ found in the low-p_T cumulative region coincides with the shape of $\psi(z)$ found in the high-p_T noncumulative region at $p_L = 70, 400$ GeV/c and $\theta_{\rm cms} \simeq 90^\circ$. These kinematic regions are quite different. This result means that self-similarity of the hadron production takes place in both the regions. Therefore, verification of the power behavior of $\psi(z)$ for the deep-cumulative pion production in $p + A$ collisions at higher p up to 2–3 GeV/c is of special interest. We expect the change of the slope parameter β of the scaling function, $\psi(z) \sim z^{-\beta}$, in this region.

CONCLUSIONS

The low-p_T data on inclusive spectra of $\pi^{\pm}$-meson production in $p+A$ collisions at incident proton momentum $p_L = 18, 58$ GeV/c and $\theta_{\rm lab} = 159°$ measured at U70 have been analyzed in the framework of z-scaling. Self-similarity of the cumulative pion production in this region was verified. Scaling function $\psi(z)$ and self-similarity parameter z for the $p + A \to \pi^{\pm} + X$ process have been constructed. They are expressed via the invariant inclusive cross section, average multiplicity density, momenta and masses of colliding and produced particles. The parameters δ_1, δ_2 are the fractal dimensions of colliding particles. They characterize self-similarity of the internal structure of proton and nucleus. They were found to be $\delta_1 = 0.5$ and $\delta_2 = A\delta_1$. The result is in agreement with similar one found for noncumulative pion production in $p + A$ collisions at high p_T [28]. The shape of the function $\psi(z)$ has been found to be the same over a wide kinematic region for different nuclei. The dependence of the fractions x_1, x_2 on the inclusive particle momentum p, atomic weight A for π^+ and π^- mesons has been studied. The $z{-}p$ plot has been used to choose the kinematic region which is most suitable to search for signatures of phase transition of the nuclear matter. The universality of the shape of $\psi(z)$ has been used to predict inclusive spectra of π^+ and π^- mesons produced in $p + A$ collisions at $p_L = 18, 58$ GeV/c and $\theta_{\rm lab} = 159°$ on Be, C, Al, Ti, Mo, W targets in the deep-cumulative region ($x_2 \gg 1/A$).

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