

Event-by-event background in estimates of the chiral magnetic effect

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(Received 17 August 2012; published 21 December 2012)

In terms of the parton hadron string dynamics (PHSD) approach—including the retarded electromagnetic field—we investigate the role of fluctuations of the correlation function in the azimuthal angle ψ of charged hadrons that is expected to be a sensitive signal of local strong parity violation. For the early time we consider fluctuations in the position of charged spectators resulting in electromagnetic field fluctuations as well as in the position of participant baryons defining the event plane. For partonic and hadronic phases in intermediate stages of the interaction we study the possible formation of excited matter in electric charge dipole and quadrupole form as generated by fluctuations. The role of the transverse momentum and local charge conservation laws in the observed azimuthal asymmetry is investigated, too. All these above-mentioned effects are incorporated in our analysis based on event-by-event PHSD calculations. Furthermore, the azimuthal angular correlations from Au + Au collisions observed in the recent STAR measurements within the Relativistic Heavy Ion Collider (RHIC) Beam Energy Scan (BES) program are studied. It is shown that the STAR correlation data at the collision energies of $\sqrt{s_{NN}} = 7.7$ and 11.5 GeV can be reasonably reproduced within the PHSD. At higher energies the model fails to describe the ψ correlation data resulting in an overestimation of the partonic scalar field involved. We conclude that an additional transverse anisotropy fluctuating source is needed which with a comparable strength acts on both in- and out-of-plane components.

DOI: [10.1103/PhysRevC.86.064907](https://doi.org/10.1103/PhysRevC.86.064907)

PACS number(s): 25.75.Ag

I. INTRODUCTION

A fundamental property of the non-Abelian gauge theory is the existence of nontrivial topological configurations in the QCD vacuum. Spontaneous transitions between topologically different states occur with a change of the topological quantum number characterizing these states and induce anomalous processes like local violation of the \mathcal{P} and \mathcal{CP} symmetry. The interplay of topological configurations with (chiral) quarks shows the local imbalance of chirality. Such a chiral asymmetry when coupled to a strong magnetic field induces a current of electric charge along the direction of the magnetic field which leads to a separation of oppositely charged particles with respect to the reaction plane [1–3].

This strong magnetic field can convert topological charge fluctuations in the QCD vacuum into a global electric charge separation with respect to the reaction plane. Thus, as argued in Refs. [1,3–5], the topological effects in QCD might be

observed in heavy-ion collisions directly in the presence of very intense external electromagnetic fields due to the “chiral magnetic effect” (CME) as a manifestation of spontaneous violation of the \mathcal{CP} symmetry. Indeed, it was shown that electromagnetic fields of the required strength can be created in relativistic heavy-ion collisions [1,6,7] by the charged spectators in peripheral collisions.

The first experimental evidence for the CME, identified via the charge asymmetry, was obtained by the STAR Collaboration at the Relativistic Heavy Ion Collider (RHIC) at $\sqrt{s_{NN}} = 200$ and 62 GeV [8–10] and confirmed qualitatively by the PHENIX Collaboration [11]. Recently, these measurements were extended, from one side, below the nominal RHIC energy down to $\sqrt{s_{NN}} = 7.7$ GeV within the RHIC Beam Energy Scan (BES) program [12] and, from the other side, preliminary results for the maximal available energy $\sqrt{s_{NN}} = 2.76$ TeV were announced from the Large Hadron Collider (LHC) [13,14]. Though at first sight, some features

of these data appear to be consistent with an expectation from the local parity violation phenomenon, the interpretation of the observed effect is still under intense discussion [15–24].

The fluctuation nature of the CME will give rise to a vanishing expectation value of a \mathcal{P} -odd observable and due to that, as proposed by Voloshin [25], the azimuthal angle two-particle correlator related to charge asymmetry with respect to the reaction plane is measured in experiments [8–10,12–14]. Accompanying these experiments hadronic estimates of the dynamical background in these experimental papers including only statistical (hadronic) fluctuations do not involve the electromagnetic field at all. The electromagnetic field—created in heavy-ion collisions—was calculated in different dynamical approaches in Refs. [1,6,7,26–28]. In two of them [7,26] calculations were carried out in comparison with the CME observable. However, in all these studies only the mean electromagnetic field was presented, being averaged over the whole ensemble of colliding nuclei.

As noted in Ref. [29] event-by-event fluctuations of the electromagnetic field in off-central heavy-ion collisions can reach rather high values comparable with the average values. The presence of large fluctuations was then confirmed in a more elaborated model in Ref. [30]. In this study—based on the parton-hadron-string dynamics (PHSD) kinetic approach—we analyze event-by-event fluctuations in the electromagnetic fields as well as in transverse momentum, multiplicity and conserved quantities and the influence of these fluctuations on physics observables relevant to measurements of the CME.

The paper is organized as follows. After a short recapitulation of the PHSD approach in Sec. II we sequentially consider the manifestation of the initial geometry fluctuations in spectator protons and participant nucleons as well as in the charged quasiparticle geometry at some later stage. These effects are relevant for fluctuations in the electromagnetic fields, the event plane orientation and the possible formation of a fluctuating electric charge dipole/quadrupole transient subsystem, respectively. Conservation of the transverse momentum and local charge is analyzed as an alternative explanation of the observed azimuthal asymmetry. In Sec. IV we discuss the role and importance of these effects in the azimuthal angle correlations and their dependence on collision energy. Our conclusions are summarized in Sec. V.

II. REMINDER OF THE PHSD APPROACH

Here we analyze the dynamics of partons, hadrons and strings in relativistic nucleus-nucleus collisions within the parton hadron string dynamics (PHSD) approach [31]. In this transport approach the partonic dynamics is based on Kadanoff-Baym equations for Green functions with self-energies from the dynamical quasiparticle model (DQPM) [32,33] which describes QCD properties in terms of ‘resummed’ single-particle Green functions. In Ref. [34], the actual three DQPM parameters for the temperature-dependent effective coupling were fitted to the recent lattice QCD results of Ref. [35]. The latter leads to a critical temperature $T_c \approx 160$ MeV which corresponds to a critical energy density of $\epsilon_c \approx 0.5$ GeV/fm³. In PHSD the parton spectral functions ρ_j

($j = q, \bar{q}, g$) are no longer δ functions in the invariant mass squared as in conventional cascade or transport models but depend on the parton mass and width parameters which were fixed by fitting the lattice QCD results from Ref. [35]. We recall that the DQPM allows one to extract a potential energy density V_p from the space-like part of the energy-momentum tensor as a function of the scalar parton density ρ_s . Derivatives of V_p with respect to ρ_s then define a scalar mean-field potential $U_s(\rho_s)$ which enters into the equation of motion for the dynamic partonic quasiparticles. Thus, one should avoid large local fluctuations in the potential V_p which indeed is solved in the parallel ensemble method by averaging the mean-field over many events. In the present study we modify the default PHSD approach by evaluating the electromagnetic fields for each event without averaging the charge currents over many (parallel) events.

The transition from partonic to hadronic degrees of freedom (d.o.f.) (and vice versa) is described by covariant transition rates for the fusion of quark-antiquark pairs or three quarks (antiquarks), respectively, obeying flavor current-conservation, color neutrality as well as energy-momentum conservation [31,34]. Since close to the phase transition the dynamical quarks and antiquarks become very massive, the formed resonant ‘prehadronic’ color-dipole states ($q\bar{q}$ or qqq) are of high invariant mass, too, and sequentially decay to the ground-state meson and baryon octets increasing the total entropy.

On the hadronic side PHSD includes explicitly the baryon octet and decuplet, the 0^- - and 1^- -meson nonets as well as selected higher resonances as in the hadron string dynamics (HSD) approach [36,37]. Note that PHSD and HSD merge at low energy density, in particular below the critical energy density $\epsilon_c \approx 0.5$ GeV/fm³.

The PHSD approach has been applied to nucleus-nucleus collisions from $\sqrt{s_{NN}} \sim 5$ to 200 GeV in Refs. [31,34] in order to explore the space-time regions of ‘partonic matter’. It was found that even central collisions at the top-SPS energy of $\sqrt{s_{NN}} = 17.3$ GeV show a large fraction of nonpartonic, i.e., hadronic or string-like matter, which can be viewed as a hadronic corona. This finding implies that neither hadronic nor only partonic ‘models’ can be employed to extract physical conclusions in comparing model results with data. All these previous findings provide promising perspectives to use PHSD in the whole range from about $\sqrt{s_{NN}} = 5$ to 200 GeV for a systematic study of azimuthal asymmetries of hadrons produced in relativistic nucleus-nucleus collisions. This expectation has been realized, in particular, in the successful description of various flow harmonics in the transient energy range [38,39].

The collision geometry for a peripheral collision is displayed in Fig. 1 in the transverse ($x - y$) plane. The reaction plane is defined as the ($z - x$) plane. The overlapping strongly interacting region (participants) has an “almond”-like shape. The nuclear region outside this ‘almond’ corresponds to spectator matter which is the dominant source of the electromagnetic field at the very beginning of the nuclear collision. Note that in the PHSD approach the particles are subdivided into target and projectile spectators and participants not geometrically but dynamically: spectators are nucleons which suffered yet no hard collision.

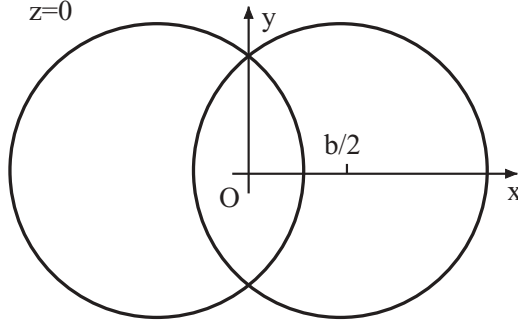


FIG. 1. The transverse plane of a noncentral heavy-ion collision. The impact parameter of the collision is denoted by b . The origin O (corresponding to $x = 0$) is referred to as the central point of the maximum overlap.

As in Refs. [6,7,29,30] the electric and magnetic fields at the relative position $\mathbf{R}_n = \mathbf{r} - \mathbf{r}_n$ are calculated according to the retarded ($t_n = t - |\mathbf{r} - \mathbf{r}_n|$) Liénard-Wiechert equations for a charge moving with velocity \mathbf{v} :

$$e \mathbf{E}(\mathbf{r}, t) = \alpha \sum_n Z_n \frac{[\mathbf{R}_n - R_n \mathbf{v}_n]}{(R_n - \mathbf{R}_n \cdot \mathbf{v})^3} (1 - v^2), \quad (1)$$

$$e \mathbf{B}(\mathbf{r}, t) = \alpha \sum_n Z_n \frac{\mathbf{v} \times \mathbf{R}_n}{(R_n - \mathbf{R}_n \cdot \mathbf{v})^3} (1 - v^2), \quad (2)$$

where the summation runs over all charged quasiparticles in the system, both spectators and participants, Z_n is the charge of the particle, and $\alpha = e^2/4\pi = 1/137$ is the electromagnetic constant. By including explicitly the participants—created during the heavy-ion reaction and being propagated in time also under the influence of the retarded electromagnetic fields—we also consider the back reaction of the particles on the retarded fields. Equations (1),(2) have singularities for $R_n = 0$ and in the calculations we regularize them by the condition $R_n > 0.3$ fm.

However, if the produced matter, after the short early-stage evolution, is in the QGP phase, the electric conductivity is not negligible. Strictly speaking our estimates of the magnetic and electric fields in Eqs. (1),(2) are strictly valid only at the early stage of the collision. At later stages we have neglected the collective electromagnetic response of the matter produced in the collision by assuming that the produced matter is ideally electrically insulating. Here, the magnetic response from the created medium is expected to become increasingly important [40] and in principle may substantially influence the time evolution of the electromagnetic fields in the QGP. In particular, a nontrivial electromagnetic response—as studied within generalized Maxwell equations including the permeability and permittivity of the QGP—can lead to a slowdown of the decrease of the magnetic field at later times of 2–4 fm/c [30,41]. It is of interest to recall that for a peripheral Au + Au collision at $\sqrt{s_{NN}} = 200$ GeV our kinetic model with the retarded electromagnetic field predicts a flattening of the strong time dependence for the magnetic field at $t \approx 1$ fm/c (see Fig. 4 in Ref. [7]). It is also noteworthy that the magnetic field strength at this time is by three orders of magnitude lower than the maximal field strength. Therefore it is not likely that

there will be a noticeable influence of the effect discussed above on observables for later times. Furthermore, we mention that according to Faraday's law a strongly decreasing magnetic field induces an electric field circulating around the direction of the magnetic field. In turn this electric field generates an electric current that produces a magnetic field pointing in the positive z direction according to the Lenz rule [42]. All these nontrivial responses of charged matter to intense electromagnetic fields are of great interest and more elaborated studies are required; however, this is beyond the aim of this present paper.

III. SOURCES OF BACKGROUND FLUCTUATIONS

A. Fluctuations in the proton spectator positions

Let us consider geometrical fluctuations in the electromagnetic field taking into account fluctuations in the position of *spectator* protons. The retarded electric and magnetic field evaluated according to Eqs. (1),(2) are presented in Fig. 2 for off-central Au + Au collisions at the collision energy of $\sqrt{s_{NN}} = 200$ GeV. The PHSD results (including contributions of all quasiparticles) are given for the time of the maximal overlap of the compressed colliding nuclei which corresponds to $t \simeq 0.05$ fm/c. As noted above the main contribution is coming from spectator protons. In peripheral collisions the average magnetic component orthogonal to the reaction plane $\langle B_y \rangle$ is dominant. The dimensionless field magnitude $e \langle B_y \rangle / m_\pi^2 \simeq 5$ and its dispersion are in a reasonable agreement (discrepancy is less than 10%) with recent calculations results within the partonic HIJING model [30]. The difference in the calculated electromagnetic field between HIJING and our PHSD approach is due to different regularization procedures used for Eqs. (1), (2). In [30] all field contributions resulting in a numerical overflow were taken away while we used a constraint on the closest distance $R_n > 0.3$ fm. The agreement between these two models demonstrates a very weak sensitivity of the results for reasonable values R_n . Our present results are consistent also with earlier calculations within the hadronic dynamics of the ultrarelativistic quantum-molecular dynamics (UrQMD) [6] and the hadron string dynamics (HSD) [7] models.

If one looks at the field variance [Fig. 2(a)] the full width of the E_y, E_x, B_x distributions is about $\sigma \sim 2/m_\pi^2$ for all transverse field components being consistent with Ref. [30]. Here, additional results are plotted also for the restricted case when the electromagnetic field is averaged over all events in the parallel ensemble as explained in the previous section. This procedure has been used before in Ref. [7]. As seen from Fig. 2 this leads to a suppression of the variance for all field distributions by a factor of about 3.

In (b) of Fig. 2 we mimic results of the schematic model in Ref. [29] considering a nuclear colliding system at the time of the maximal overlap as an infinitely thin disk. This was simulated numerically by an artificial shift of the position of the longitudinal components of all protons at this moment to the plane $z = 0$. As is seen in Fig. 2(b) all field distributions indeed increase in width by a factor of about two. A direct comparison of our results to those of Ref. [29] gives a factor

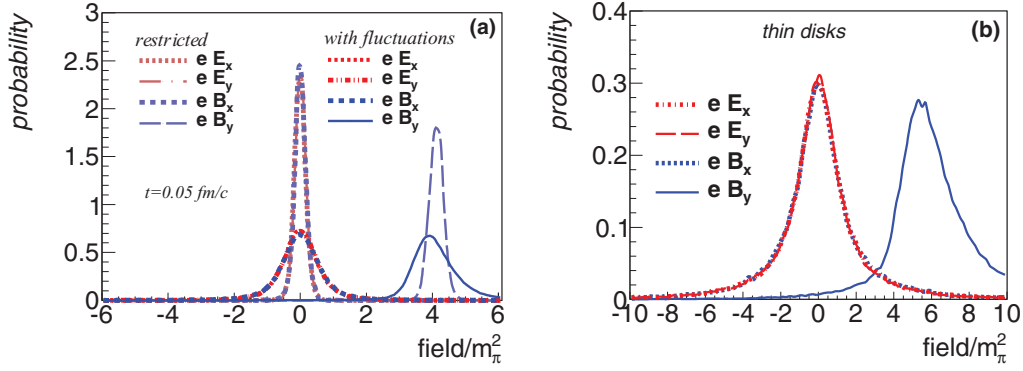


FIG. 2. (Color online) Probability densities for electromagnetic fields at the central point of off-central Au + Au collisions for $\sqrt{s_{NN}} = 200$ GeV at the time of maximal overlap as emerging from event-by-event calculations. The results are given for the PHSD model (a) and for the schematic model with infinitely thin nuclei as in Ref. [29] (b).

of three or even more. This finding completely coincides with the results of Ref. [30] as to both the value of the width and its origin.

The estimated strength of the electromagnetic fields provides no information about their action on the quasiparticle transport. Let us look at the early time dynamics in more detail and introduce a momentum increment $\Delta \mathbf{p}$ as a sum of the mean particular increases of the quasiparticle momentum $d\mathbf{p}$ due to the action of the electric and magnetic forces,

$$\mathbf{F}_{em} = e\mathbf{E} + (e/c)\mathbf{v} \times \mathbf{B}, \quad (3)$$

during the short time interval at the expense of the given source,

$$\Delta \mathbf{p}(t) = \sum_{t_i}^t \langle d\mathbf{p}(t_i) \rangle. \quad (4)$$

Equation (4) is considered on an event-by-event basis and for each event the mean momentum increase during a time-step $\langle d\mathbf{p}(t_i) \rangle$ is calculated over all particles involved. In Fig. 3 the average momentum change of forward moving quarks ($p_z > 0$) is shown for three components of the electromagnetic force at $\sqrt{s_{NN}} = 200$ GeV. Note the different scales for the solid lines in Fig. 3 that give the net momentum change at this energy. It is a remarkable fact that the transverse electric and magnetic components compensate each other almost completely.

Two remarks are in order: First, due to the linearity of the electromagnetic force (3) with respect to the electric and magnetic field, one should not expect a difference in quark transport calculations with and without taking into account electromagnetic field fluctuations. This was demonstrated for quasiparticles earlier in terms of the HSD model [26]. Second, if transverse fluctuations are characterized by the average strength of the fields, $\langle |E_{x,y}| \rangle$ and $\langle |B_{x,y}| \rangle$, certain equalities between components like $\langle |E_x| \rangle \approx \langle |E_y| \rangle \approx \langle |B_x| \rangle$ —as numerically obtained in Ref. [29] and confirmed in Ref. [30]—imply that similar equalities should hold for the fluctuations. Indeed, similar relations follow from our PHSD calculations, see Fig. 2(a) where the increment functions for appropriate field components practically coincide. We emphasize again that the PHSD transverse field components are not only of comparable strength but their action on the quarks [see Eq. (3)]

approximately compensate each other. One should note that this is the compensation effect rather than the short lifetime of the electromagnetic interaction which leads to a very weak sensitivity of observables as has been demonstrated recently in terms of the hadronic HSD transport model in Ref. [26]. For a quasiparticle moving along the trajectory $x = x(t)$, this

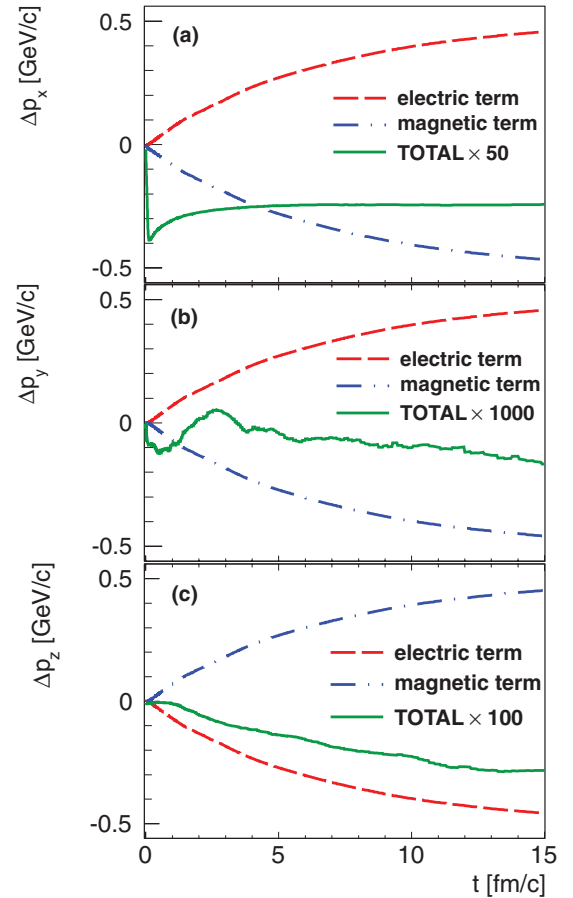


FIG. 3. (Color online) Time dependence of the momentum increment of forward moving ($p_z > 0$) partons due to the electromagnetic field created in Au + Au ($\sqrt{s_{NN}} = 200$ GeV) collisions with the impact parameter $b = 10$ fm.

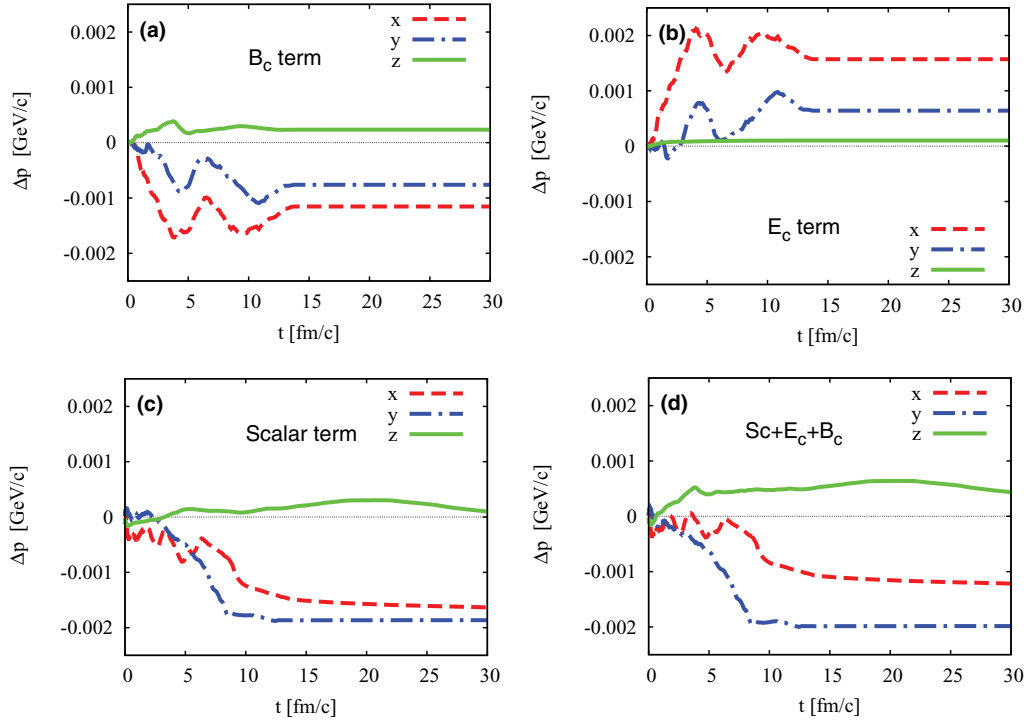


FIG. 4. (Color online) Time dependence of the momentum increment from ‘electric’ E_c and ‘magnetic’ B_c (a),(b) partonic field components, the scalar field and total momentum increment (c),(d) for forward moving ($p_z > 0$) positively charged quarks and $p > 0$ as a function of time t . The system is Au + Au (at $\sqrt{s_{NN}} = 200$ GeV) for the impact parameter $b = 10$ fm.

compensation in a simplified 1D case can be illustrated by a short calculation as

$$eE = -e \frac{\partial A}{\partial t} \sim -e \frac{\partial A}{\partial x} \frac{dx}{dt} \sim -eBv, \quad (5)$$

i.e., the action of the electric and magnetic transverse components is roughly equal and directed oppositely.

The important advantage of the PHSD approach relative to hadron-string models is the inclusion of partonic degrees of freedom. In particular, the involved partonic fields (of scalar and vector type) showed up to be essential to describe the elliptic flow excitation function from lower SPS to top RHIC energies [38,39] and to be a key quantity in analyzing the CME.

The evolution of momentum increments for partonic forces is presented in Fig. 4 for off-central Au + Au collisions at $\sqrt{s_{NN}} = 200$ GeV. It is seen that (marked by the subscript ‘c’) the transverse ‘electric’ E_c and ‘magnetic’ field B_c of the partonic field components [Fig. 4(a),(b)] almost compensate each other. The z component is practically vanishing and for $t \gtrsim 8$ fm/c all quark increments stay roughly constant, i.e., the quark phase ends here. The final action of the partonic forces is defined by the sum of the forces (d) which is dominated by the scalar one.

Apart from the average forces (momentum increments) the fluctuations of the forces are of further interest. As seen from Fig. 5 the distribution in the quark momentum deviation $\delta \mathbf{p} = \mathbf{p} - \langle \mathbf{p} \rangle$ in case of scalar forces is well collimated with respect to the average trajectory $\langle \mathbf{p} \rangle$ presented in Fig. 4 but its width increases by about a factor of three when proceeding

from $t = 0.05$ to 3.0 fm/c [Fig. 5(a)–5(c)]. This spread is slightly larger in the x component since the derivatives of the scalar mean-field are higher in the x than in the y direction. The influence of the electromagnetic force on quarks and charged pions is visible more clearly [Fig. 5(d)–(i)] in the early time corresponding to the maximal overlap of the colliding nuclei ($t = 0.05$ fm/c) when the created electromagnetic field is maximal. Here, the $\langle \delta p_x \rangle$ component is shifted for quarks (d)–(f) and even more for mesons (g)–(i). This shift decreases in time and disappears for $t = 3$ fm/c; at this time the deviation distributions for all three components of the electromagnetic force are close to a δ function.

Some general considerations on parity violation in heavy-ion reactions are in order here: Since the magnetic field is odd under time reversal (or equivalently, under the combined charge conjugation and parity \mathcal{CP} transformation), the time reversal symmetry of a quantum system is broken in the presence of an external magnetic field. A magnetic field \mathbf{B} can also combine with an electric field \mathbf{E} to form the Lorentz invariant $(\mathbf{E} \cdot \mathbf{B})$ which changes the sign under a parity transformation. In the normal QCD vacuum with its spontaneously broken chiral symmetry the leading interaction involves the invariant $(\mathbf{E} \cdot \mathbf{B})$ which enters, e.g., into the matrix element that mediates the two-photon decay of the neutral pseudoscalar mesons. In the deconfined chirally symmetric phase of QCD, the leading interaction term is proportional to $\alpha_s (\mathbf{E} \cdot \mathbf{B})(\mathbf{E}^a \cdot \mathbf{B}^a)$, where \mathbf{E}^a and \mathbf{B}^a denote the chromoelectric and chromomagnetic fields, respectively, and α_s is the strong QCD coupling. Both interactions are closely related to the electromagnetic axial anomaly, which in turn relates the

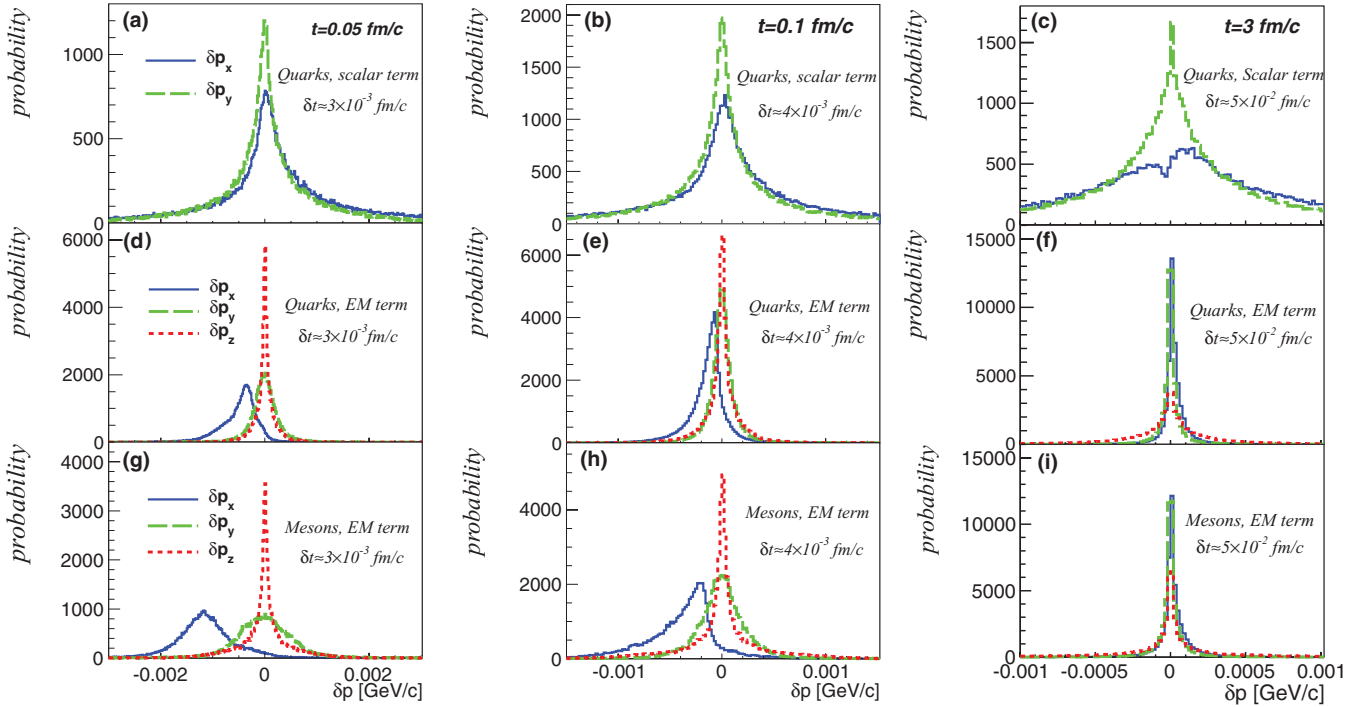


FIG. 5. (Color online) Probability distribution of the momentum increment $\delta\mathbf{p} = \mathbf{p} - \langle\mathbf{p}\rangle$ during the time step δt for forward moving positively charged quasiparticles at times $t = 0.05, 0.1$, and 3 fm/c. The distribution emerging from the quark scalar potential is shown in the upper panels (a),(b),(c) whereas the distribution stemming from the EM field of quarks and mesons are displayed in the middle (d),(e),(f) and bottom (g),(h),(i) panels, respectively. The calculations have been performed for off-central Au + Au collisions at $\sqrt{s_{NN}} = 200$ GeV and impact parameter $b = 10$ fm in the PHSD model.

divergence of the isovector axial current to the pseudoscalar invariant of the electromagnetic field (see Ref. [43]). The evolution of the electromagnetic invariant $\mathbf{E} \cdot \mathbf{B}$ is shown in Fig. 6. The case of Au + Au ($\sqrt{s_{NN}} = 200$ GeV) collisions at impact parameter $b = 10$ fm is considered. As seen from Fig. 6 the electromagnetic invariant ($\mathbf{E} \cdot \mathbf{B}$) is nonzero only in the initial time $t \lesssim 0.5$ fm/c where the ($\mathbf{E} \cdot \mathbf{B}$) distribution is quite irregular and its nonzero values correlate well with the location of the nuclear overlap region. For later times this electromagnetic invariant vanishes in line with the electric field space-time distributions [7]. Note that the quantities plotted in Fig. 6 are dimensionless and the scaling factor m_π^4 [GeV⁴] is quite small.

One should note that in addition to the strong electromagnetic fields [1,6] present in noncentral collisions, very strong color electric \mathbf{E}^a and color magnetic \mathbf{H}^a fields are produced in the very beginning of these collisions as shown in the non-Abelian field theory [44]. These fields can be characterized by a gluon saturation momentum Q_s and the time $\sim 1/Q_s$. Both fields are parallel to each other and directed along the z axis. This leads to a nonzero topological charge $Q \sim (\mathbf{E}^a \cdot \mathbf{B}^a) \neq 0$. Since gauge fields with $Q \neq 0$ generate chirality, they also can induce electromagnetic currents along a magnetic field [45] resulting in the CME. Though a large amount of topological charge might be produced through the mechanism of sphaleron transitions, the primary mechanism for topological charge Q generation at the early stage is by fluctuations of color electric and magnetic fields. The decay of these fields is essentially governed by the non-Abelian

dynamics of the glasma [44,46] which ultimately produces the QGP (close to equilibrium). Unfortunately, this possible mechanism for the CME is beyond the potential of the PHSD model used. We thus may speculate about but not prove this mechanism.

B. Fluctuations in the position of participant nucleons

As noted above (see Fig. 1), the interaction region after averaging over many events has an almond-like shape; the averaged spatial initial asymmetry of the participant matter is symmetric with respect to the reaction plane. Actual collision profiles, however, are not smooth and the symmetry axis in an individual event is tilted due to fluctuations (cf. Fig. 7). The geometry fluctuations in the location of the *participant* nucleons lead to fluctuations of the participant plane (PP) from one event to another, rendering larger coordinate space eccentricities which due to pressure gradients are translated into elliptic flow for the final state particles. Thus, the system of the elliptical almond-like shape expands predominantly along the minor axis.

Depending on the location of the participant nucleons in the colliding nuclei at the time of the collision, the actual shape of the overlap area may vary. As is seen from Fig. 7, due to fluctuations the overlap area in a single event can have, for example, a rotated triangular rather than an almond shape. Note that an almond shape is regained by averaging over many events for the same impact parameter. However, in experiment the collective flows are measured with

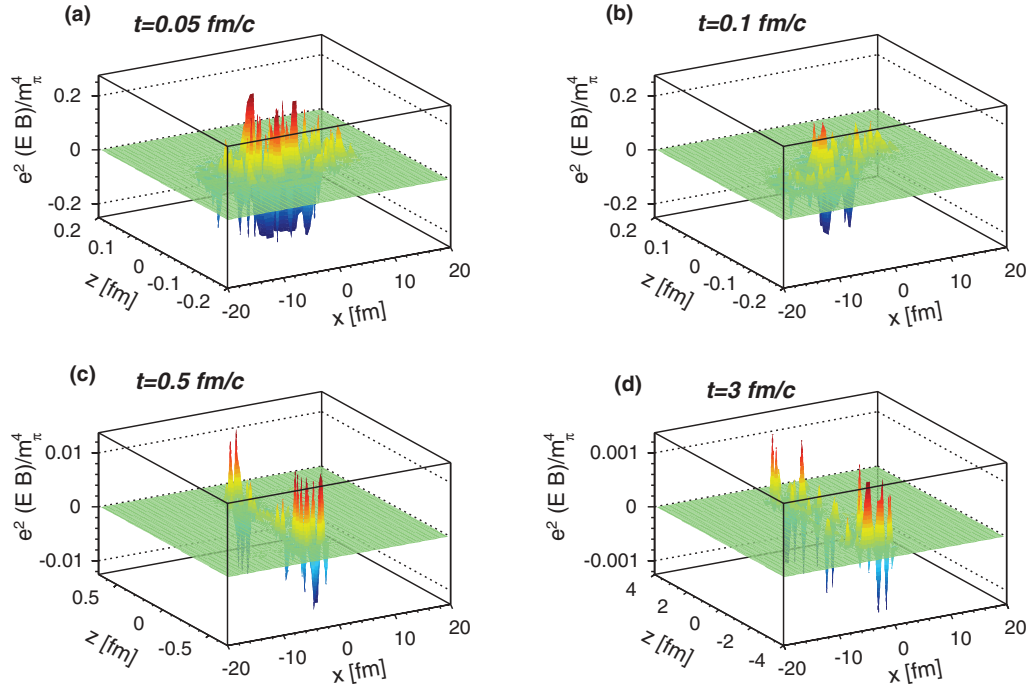


FIG. 6. (Color online) Space-time evolution of the scalar product of electric and magnetic fields ($\mathbf{E} \cdot \mathbf{B}$) for Au + Au reactions at impact parameter $b = 10$ fm and $\sqrt{s_{NN}} = 200$ GeV. Note the different scales along the z axis.

respect to a third plane, the so-called *event* plane defined by observable charged participants in momentum space through the harmonic/multipole analysis. More precisely, the flow coefficients v_n are defined as the n th Fourier harmonic of the particle momentum distribution with respect to the particular momentum event plane Ψ_n ,

$$\langle v_n \rangle = \langle \cos[n(\psi - \Psi_n)] \rangle, \quad (6)$$

where $\psi = \arctan(p_y/p_x)$ is the azimuthal angle of the particle momentum \mathbf{p} in the c.m. frame and angular brackets denote a statistical average over many events.

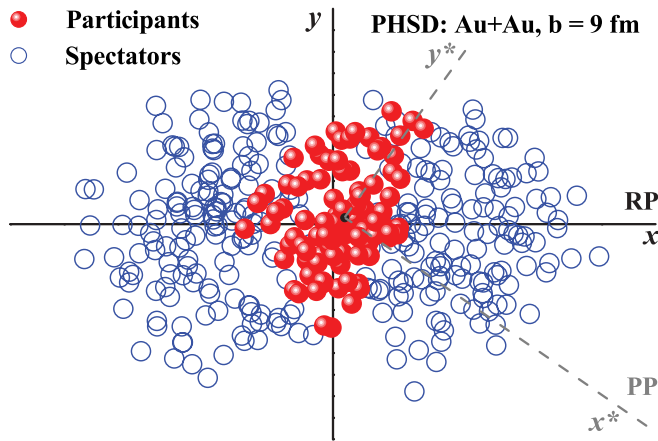


FIG. 7. (Color online) Projection of a single peripheral Au + Au (200 GeV) collision on the transverse plane. Spectator and participant nucleons are plotted by empty and filled circles, respectively. The reaction plane (RP) projection corresponds to x axis. Transverse axes of the participant plane (PP) are marked by stars (x^* , y^*).

One should note that all azimuthal correlations are not only due to the collective flow. The early-time two-particle spatial correlations probe both the event geometry (fluctuating in individual events) and genuine local pair correlations referred to as ‘nonflow’ correlations. The Fourier decomposition (6) is not enough to disentangle these two contributions. A possible solution of the connection between flow fluctuations and initial state correlations is given by the cumulant expansion method [47] using two- and four-particle correlation measurements of the harmonic flow coefficients. However, this method is beyond the scope of the present study.

The distributions in the event plane angle for different harmonics are shown in Fig. 8 for the freeze-out case. All distributions are symmetric with respect to the point $\Psi_n = 0$ which corresponds to the true reaction plane. As is seen, the event plane angle Ψ_n determined from the “ n th” harmonic is in the range $0 \leq \Psi_n < 2\pi/n$ and fluctuations of several lowest order harmonics have comparable magnitudes. Inside this region Ψ_1 has two maxima at $\Psi_n = 0$ and π corresponding to forward-backward emission. The even components Ψ_2 , Ψ_4 have a rather prominent maximum for $\Psi_n = 0$ indicating the local nature of fluctuations, but the odd harmonics Ψ_3 , Ψ_5 are practically flat. This may be easily understood since the odd moments of the spatial anisotropy purely originate from fluctuations while the even ones are combined effects of fluctuations and geometry. As a consequence, if one defines the spatial anisotropy parameters with respect to the pre-determined reaction plane, the event-averaged ones vanish for all odd moments but not for the even moments.

The histograms in Fig. 8 are calculated from a sample of 3×10^4 events taking into account magnetic and electric field fluctuations. Similar calculations without fields are shown in

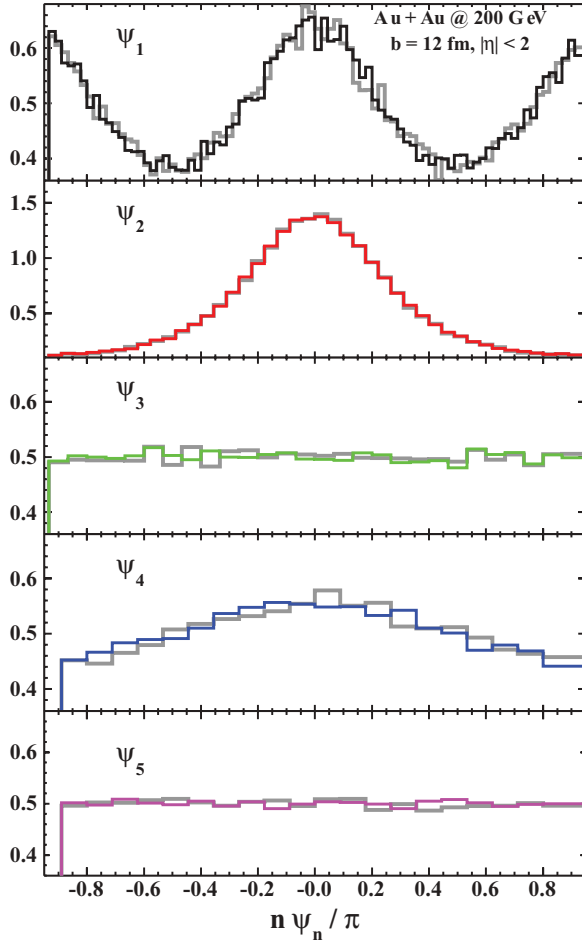


FIG. 8. (Color online) Distribution in the event plane angle for different harmonics Ψ_n calculated with retarded magnetic and electric fields. Grey histograms show the results for respective calculations without fields.

the same figure by the grey histograms which are hard to distinguish from the previous ones. In other words, there is no additional “tilting” effect by electromagnetic fields as expected in Refs. [29,30]. This is due to the compensation of the transverse electromagnetic components as explained above.

C. Two-particle angular correlations

An experimental signal of the local spontaneous parity violation is a charged particle separation with respect to the reaction plane [48]. It is characterized by the two-body correlator in the azimuthal angles,

$$\begin{aligned} \gamma_{ij} &\equiv \langle \cos(\psi_i + \psi_j - 2\Psi_{RP}) \rangle \\ &= \langle \cos(\psi_i - \Psi_{RP}) \cos(\psi_j - \Psi_{RP}) \rangle \\ &\quad - \langle \sin(\psi_i - \Psi_{RP}) \sin(\psi_j - \Psi_{RP}) \rangle, \end{aligned} \quad (7)$$

where Ψ_{RP} is the azimuthal angle of the reaction plane defined by the beam axis and the line joining the centers of the colliding nuclei and subscripts of γ_{ij} represent the signs of electric charges being positive or negative. The averaging in Eq. (7) is carried out over the whole event ensemble and cos and

sin terms in Eq. (7) correspond to out-of-plane and in-plane projections of γ_{ij} .

As was proposed in Refs. [17,19] and more elaborated in Ref. [22], a possible source of azimuthal correlations among participants is the conservation of the transverse momentum which might give rise to a contribution comparable with the measured CME. Transverse momentum conservation (TMC) introduces back-to-back correlations for particle pairs because they tend to balance each other in transverse momentum space. A large multiplicity of particles will dilute the effect of these two-particle correlation. Furthermore, this correlation should be stronger in plane than out of plane due to the presence of the elliptic flow. Nevertheless, the TMC provides a background for the CME that should be properly quantified.

From quite general considerations—making use of the central limit theorem and describing particles thermodynamically—one can derive the following simple expression for the two-particle correlator [22]:

$$\gamma_{ij} = \langle \cos(\psi_i + \psi_j) \rangle = -\frac{v_2 \langle p_t \rangle_{\text{acc}}^2}{N \langle p_t^2 \rangle_{\text{full}}}, \quad (8)$$

where $N = N_+ + N_0 + N_-$ is the total number of all produced particles (in full phase space). At $\sqrt{s_{NN}} = 200$ GeV it can be approximated as $N \approx (3/2) N_{ch} \approx 21 N_{\text{part}}$ [22] where N_{part} is calculated dynamically in our model as well as the momentum-dependent factors for full phase space and the ratio to the measured accepted phase space. It is of interest to note that the proportionality of the CME to the elliptic flow v_2 seen in Eq. (8) follows also from more elaborated considerations. In particular, the chiral magnetic effect in the hydrodynamic approach and in terms of a holographic gravity dual model (see Ref. [49]) predicts a linear dependence of the CME on v_2 with more sophisticated coefficients which depend on the axial anomaly coefficient and the axial chemical potential as well as on dynamics of fluids through the particle density, baryon chemical potential and pressure.

Experimentally, the same-sign correlator γ_{ss} is defined as the average of γ_{++} and γ_{--} by assuming that the momentum balance is shared equally among the charges

$$\gamma_{ss} = \frac{1}{2}(\gamma_{++} + \gamma_{--}) = -\frac{v_2 \langle p_t \rangle_{\text{acc}}^2}{N \langle p_t^2 \rangle_{\text{full}}}, \quad (9)$$

where the subscripts “full” and “acc” imply that average values should be calculated in the “full” phase space or in the proper “acceptance region”, respectively. In practice, only a subset of particles is measured. In this case some of the momentum balance stems from unmeasured particles and one might expect $\gamma_{ss} \ll v_2/N$ [19]. In the STAR experiment [51] tracks were measured for the central two units of rapidity. However, the initial colliding beams approached with ± 5.5 units of rapidity and more than 50% of the charged particles tracks have rapidities outside the STAR acceptance. These particles can serve as a source of momentum, which may quench the momentum conservation condition thus reducing the magnitude of γ_{ss} . However, the transverse momentum of a given track is more likely to be balanced by neighboring particles, which have similar rapidities. This is particularly true when considering the components of the momenta responsible

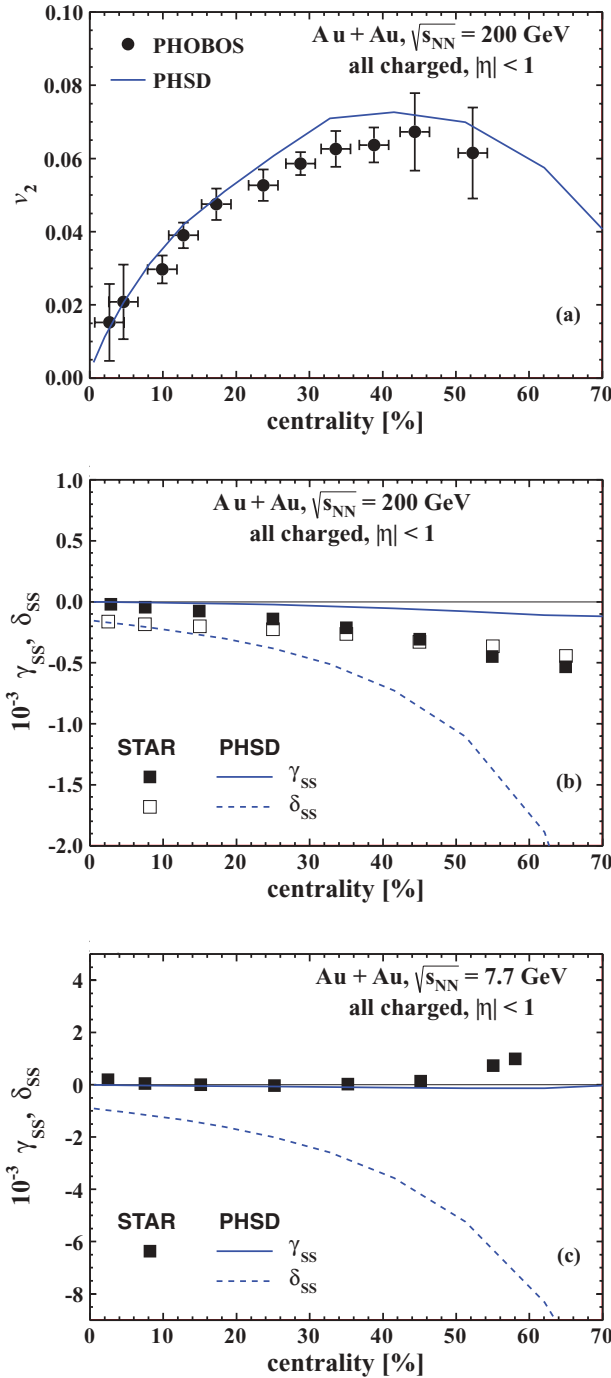


FIG. 9. (Color online) PHSD centrality dependence of the elliptic flow (a) and angular correlators γ_{ss} and δ_{ss} of charged particles from Au + Au at $\sqrt{s_{NN}} = 200$ (b) and 7 (c) GeV from the transverse momentum conservation according to Eqs. (9) and (10). The experimental data points for v_2 and γ_{ss} , δ_{ss} are from Refs. [50] and [12], respectively.

for elliptic flow. We conclude that this effect should be more essential for lower collision energy.

The direct comparison of the momentum conservation effect (9) on the CME observable is presented in Fig. 9 for the top RHIC energy. The total (rather than transverse) momentum conservation is inherent in the PHSD model. In the actual

calculations the experimental acceptance $p_t > 200$ MeV/c is taken into account; as seen from the upper part of Fig. 9 the centrality dependence of the elliptic flow $\langle v_2 \rangle$ for charged particles is rather well reproduced by PHSD. However, the experimental same-sign correlator γ_{ss} is underestimated substantially. We note that the experimental acceptance essentially influences the momentum-dependent ratio $\langle p_t \rangle_{acc}^2 / \langle p_t \rangle_{full}^2$. In reality the difference in γ_{ss} should be even larger as discussed above. This point is in agreement with the full HSD calculation of the hadronic background within the CME studies in Ref. [26].

A similar analysis for the lower energy $\sqrt{s_{NN}} = 7.7$ GeV is presented in the panel (c) of Fig. 9. Unfortunately, measured data for the centrality dependence of the elliptic flow are not available at this energy but the PHSD calculated average $\langle v_2 \rangle$ for minimum bias collisions is only slightly below the experiment [39] due to neglecting a baryon mean-field potential (see also the end of Sec. III D). The calculated correlation γ_{ss} strongly differs from the measured values having even the opposite sign. One should note that in this case the same and opposite sign components are almost equal to each other (i.e., there is no charge separation effect). This observation is also nicely reproduced within the HSD model at this energy (cf. Ref. [26]).

It is of further interest to consider the average cosine of the transverse angle difference which is independent of the reaction plane

$$\delta_{ij} \equiv \langle \cos(\psi_i - \psi_j) \rangle = -\frac{\langle p_t \rangle_{acc}^2}{N \langle p_t^2 \rangle_{full}}, \quad (10)$$

where the last equality is obtained from the transverse momentum conservation [22]. As follows from the comparison between Eqs. (8) and (10), the correlator δ_{ij} differs from γ_{ij} only by the elliptic flow coefficient v_2 and is expected to be more sensitive to the TMC. As one can see from Fig. 9 this estimate of δ_{ss} is too large and hardly consistent with appropriate experimental data from Fig. 18.

Thus, the considered angular correlation γ_{ss} is generated by a combination of momentum conservation, which causes particles to be preferably generated in the opposite direction, and elliptic flow which gives more particles in the $\pm x$ direction than in the $\pm y$ direction. However, this source is by far not able to explain the observed pion asymmetry in the angular correlation. In addition, the considered TMC is blind to the particle charge and cannot disentangle same-sign and opposite-sign pair correlations.

D. Electric charge fluctuations in the transient stage

The almond-like fireball created in the early collision phase then expands in an anisotropic way, however, the spatial anisotropy is reduced with increasing time. In this transient stage the electromagnetic field is strongly reduced since the spectator matter is flying away from the formed fireball. The pressure gradients act predominantly in the reaction plane resulting in elliptic flow v_2 .

Strong interactions in this phase might produce significant fluctuations in energy density (temperature), transverse momentum, multiplicity and conserved quantities such as the net

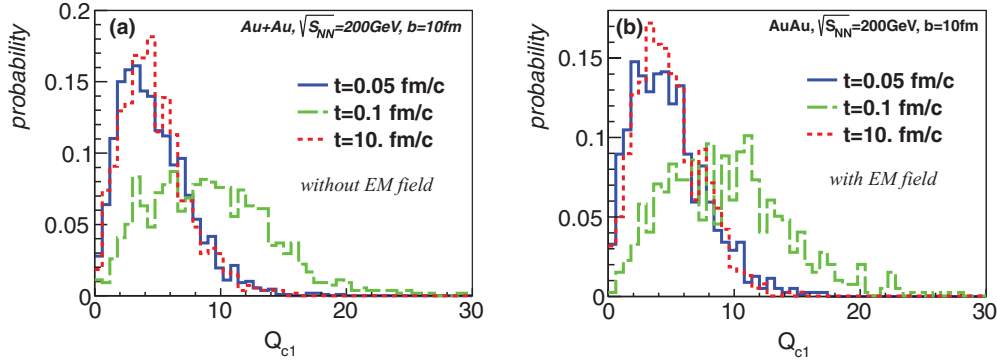


FIG. 10. (Color online) Probability distribution in the magnitude Q_{c1} of the generated electric dipole at freeze-out for all partons. The panels (a) and (b) correspond to calculations without and with the electromagnetic field, respectively. The system is Pb + Pb at $\sqrt{s_{NN}} = 200$ GeV at impact parameter $b = 10$ fm.

charge. In the plasma phase in a magnetic field an electric quadrupole can be formed due to chiral anomaly and as a signal of that the elliptic flow difference between π^+ and π^- mesons is predicted $v_2(\pi^+) < v_2(\pi^-)$ [52]. Certainly, to check that the influence of hadronic transport on observables should be taken into account.

Furthermore, the CME [1] predicts that in the presence of a strong electromagnetic field at the early stage of the collision the sphaleron transitions in a hot and dense QCD matter induce a separation of charges along the direction of the magnetic field which is perpendicular to the reaction ($x - z$) plane. This charge separation results in the formation of an electric dipole in momentum space which breaks parity. Being interested essentially in the quark phase, we investigate in this subsection to what extent such an electric dipole can be generated by background statistical and electromagnetic field fluctuations.

Let us quantify the dipole defining the plane \hat{Q}_{c1} of the quark distribution in the transverse momentum space. The magnitude Q_{c1} and azimuthal angle Ψ_{c1} of this vector can be determined in a given event as follows:

$$\begin{aligned} Q_{c1} \cos \Psi_{c1} &= \sum_i q_i \cos \psi_i, \\ Q_{c1} \sin \Psi_{c1} &= \sum_i q_i \sin \psi_i, \end{aligned} \quad (11)$$

where the summation runs over all charged particles in the event with the electric charge q_i and azimuthal angle ψ_i of each particle. Note that Eq. (11) describes the dipole shape of charged particles (quarks or hadrons) without any reference to the charge separation.

As seen from Fig. 10, the average magnitude of the electric dipole \bar{Q}_{c1} at the moment of the maximum nuclear overlap ($t = 0.05$ fm/c) is about 4 charge units with dispersion $\sigma_{Q1} \approx 2$. At this moment the system is in the quark phase having on average an almond-like shape and therefore is expanding preferentially along the x axis. Note that according to Eq. (11) quark net electric charges with $|q_i| < 1$ are considered. Thus, the number of quarks involved in the dipole is large. In the next step ($t = 0.1$ fm/c) of the expansion stage (dot-dashed line in Fig. 10) the Q_{c1} distribution is getting

broader with a noticeable increase of \bar{Q}_{c1} . At $t = 10$ fm/c the quark-gluon phase transforms predominantly into the hadronic phase through the dynamical coalescence mechanism and the Q_{c1} distribution becomes narrower again. The influence of the electromagnetic field on this evolution is very weak [compare the top (a) and bottom (b) panels in Fig. 10].

The whole evolution of the electric dipole is seen more clearly in the 3D representation in Fig. 11. Indeed, the Q_{c1} distribution has a pronounced peak formed shortly after the collision, then the magnitude of the Q_{c1} charge distribution is minimal during about 3 fm/c to testify that a large-in-charge subsystem is formed. After that the Q_{c1} distribution is getting narrower because during the expansion the Q_{c1} value slightly decreases due to parton hadronization; the maximum of the probability distribution increases and then stabilizes after $t \approx 10$ fm/c.

Similarly to the dipole, one can define a charged quadrupole formed in heavy-ion collisions as follows:

$$\begin{aligned} Q_{c2} \cos 2\Psi_{c2} &= \sum_i q_i \cos 2\psi_i, \\ Q_{c2} \sin 2\Psi_{c2} &= \sum_i q_i \sin 2\psi_i. \end{aligned} \quad (12)$$

Characteristics of the time evolution of the electric dipole and quadrupole (formed in semicentral Au + Au at $\sqrt{s_{NN}} =$

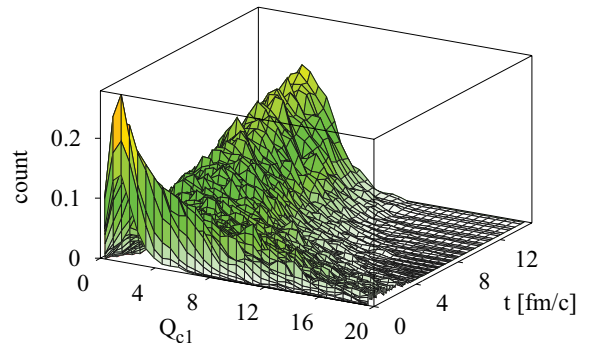


FIG. 11. (Color online) Electric dipole evolution of charged partons in $Q_{c1} - t$ presentation for Au + Au collisions at $\sqrt{s_{NN}} = 200$ GeV with taking into account the electromagnetic field.

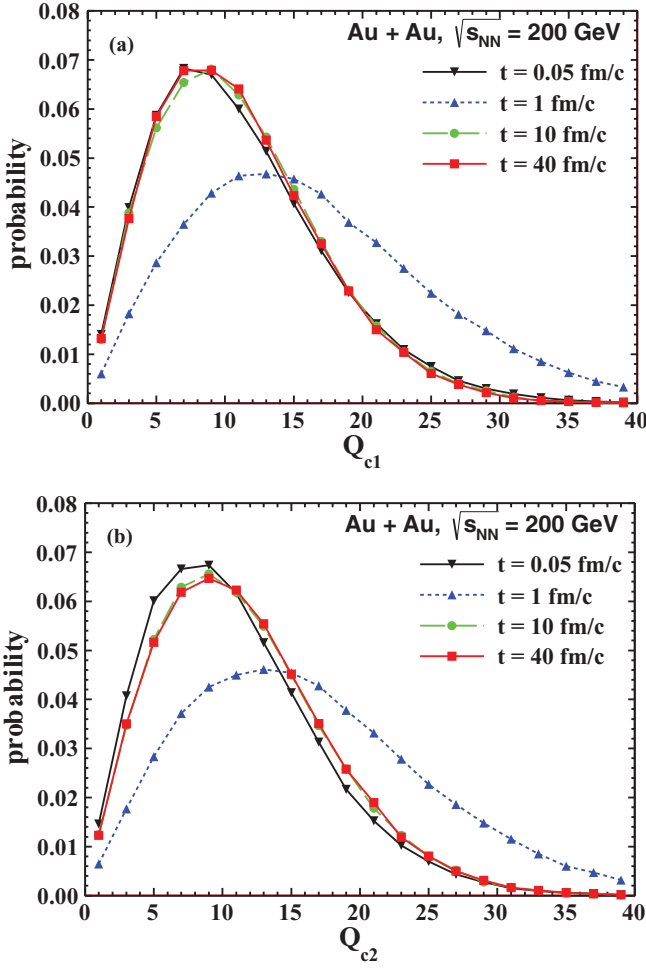


FIG. 12. (Color online) Distribution in the magnitude of the charge dipole (a) and quadrupole (b) calculated for charged particles at times $t = 0.05, 1, 10$, and 40 fm/c. The system is Au + Au at $\sqrt{s_{NN}} = 200$ GeV and impact parameter $b = 10$ fm.

200 GeV collisions) for all (parton and hadrons) charged quasiparticles in the midrapidity range are presented in the next two figures. The Q_{c1} minimum observed in the pure partonic case (see Figs. 10 and 11) survives also in this case. The magnitude of the $n = 2$ quadrupole harmonics (presented in Fig. 12) is close to that for the dipole $n = 1$, i.e., $Q_{c2} \approx Q_{c1}$ and their maximal values extend to values of 30–40. As is seen from Fig. 13, the distributions in the reaction plane angle for the electric quadrupole Ψ_{c2} are rather flat during the whole evolution while the electric dipole angle Ψ_{c1} distribution is flat only in the partonic phase (see $t = 0.05$ fm/c in Fig. 13) but in the hadronic phase the distribution resembles that for the directed flow (c.f. Fig. 8). The main axis of Ψ_{c1} and Ψ_{c2} can randomly be parallel or antiparallel to the minor axis of the almond. Like in the dipole case (cf. Fig. 10) the electromagnetic field has no sizable influence on the characteristics of the electric quadrupole. When the collision energy $\sqrt{s_{NN}}$ decreases the behavior of the dipole and quadrupole distributions in the magnitudes Q_{c1} and Q_{c2} and the angle practically do not change besides some structure in the Ψ_{c1} distribution. As seen from Fig. 14 at

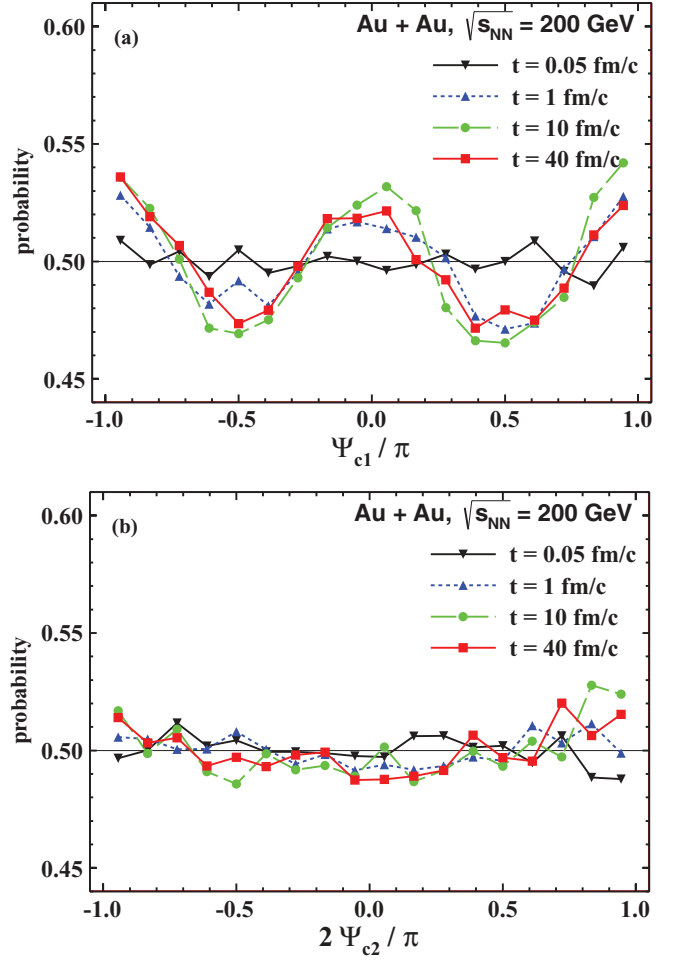


FIG. 13. (Color online) The same as in Fig. 12 but for the distribution in the event plane angle of the charge dipole (a) and quadrupole (b) at times $t = 0.05, 1, 10$, and 40 fm/c.

$\sqrt{s_{NN}} = 11.5$ GeV back-to-back correlations—as specific for the directed flow—are manifested. This is mainly due to the

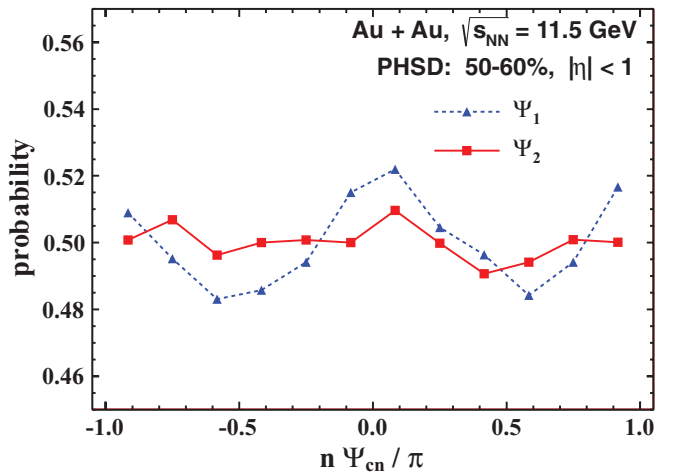


FIG. 14. (Color online) Angular distribution of charged particles for the charge quadrupole (solid line) and dipole (dashed) subsystems calculated for Au + Au ($\sqrt{s_{NN}} = 11.5$ GeV) collisions.

proton contribution which becomes noticeable at low collision energy.

Thus, the statistical fluctuations of “normal” matter in the presence of the retarded electromagnetic field do not result in a sizable formation of a deformed subsystem of dipole- or quadrupole-shape during the evolution of the heavy-ion collision.

We point out, however, that such subsystems might be formed in nontrivial topological systems due to the chiral anomaly effect. In particular, it happens when a quark experiences both a strong magnetic field and a topologically nontrivial gluonic field such as an instanton [53]. The inherent asymmetry—when both instanton and magnetic field are present—can lead to the development of an electric dipole moment. Physically, it can be understood as the result of two competing effects: the spin projection produced by a magnetic field and the chirality projection produced by an instanton field. Such a consideration is beyond the scope of our present microscopic study.

It is of interest that the axial anomaly in a strong external magnetic field induces not only the CME but also the separation of the chiral charge. The coupling of the density waves of electric and chiral charge results in the ‘chiral magnetic wave’ and can induce a static quadrupole moment of the electric charge density [52]. This chiral magnetic wave results in the degeneracy between the elliptic flows of positive and negative pions leading to $v_2(\pi^-) > v_2(\pi^+)$, which was estimated theoretically on the level of $\sim 30\%$ for midcentral Au + Au collisions at $\sqrt{s_{NN}} = 11$ GeV [52]. Our PHSD calculations give about 6% which is quite comparable with the recently measured value of 10% [54] and essentially smaller than the prediction of Ref. [52]. Noteworthy that the v_2 degeneracy in the PHSD version used is only due to different elastic and inelastic cross sections for π^+ and π^- mesons but without taking into consideration the (small) mean-field pion-nucleus potential. The elliptic flow analysis of the difference between particles and antiparticles (including kaons and baryons alongside with pions) shows that this difference is coming mainly from the hadronic mean-field potential [55]. Recently these v_2 data have been also successfully explained in terms of a hybrid model, which combines the fluid dynamics of a fireball evolution with a transport treatment of the initial and final hadronic states [56]. Therefore, there is not much room for the contribution from a transient charged quadrupole due to the chiral magnetic wave.

E. Charge balance functions

In the formation of the charged dipole and quadrupole there is no information about a possible charge separation which could result in an electric driving force. In principle such information can be provided by the balance function which is based on the idea that charge is locally conserved when particles are produced pair-wise. In the subsequent expansion of the system and rescattering of the charge carriers, which in principle can be hadronic or partonic, the balancing partners are then spread out within some finite distance to each other. The original correlation in space-time transforms into a correlation in momentum space in the final hadronic

emission profile. Therefore, the motion of the balancing partners suffers from the collective expansion of the system and diffusion due to the collisions with other particles. The study of charge-balance correlations hence gives insight into the production and diffusion of charge. In particular, it is expected that the balance function is sensitive to the delayed transition of the quark-gluon phase to a hadronic phase [57].

Thus, whenever a positive charge is created, a negative charge arises from the same point in space-time and both particles then tend to be focused in the same rapidity and azimuthal angle by collective flow. This results in a correlation between positive and negative charges, i.e., for every positively charged particle emitted at an angle ψ_+ , there tends to be a negatively charged particle emitted with $\psi^- \approx \psi^+$ and similar rapidity. Charge balance functions [57] represent a measure of such correlations, and have already been investigated as a function of relative rapidity for identified particles and for relative pseudorapidity η for nonidentified particles [51,58]. Generally, the balance function $B(p_a|p_b)$ is a six-dimensional function of the particle momenta. In the context of studies of the separation of balancing charges, the discussion is reduced to the difference $(\mathbf{p}_1 - \mathbf{p}_2)$. In particular we will focus on the charge balance function in relative pseudorapidity $\delta\eta$, i.e., $B(p_a|p_b) \rightarrow B(\delta\eta, \eta_w)$ and similarly for the azimuthal angle ψ [59].

Charge balance functions are constructed in such a way that like-sign subtractions statistically isolate the charge balancing partners,

$$B(\delta\eta, \eta_w) = \frac{1}{2} \left(\frac{N_{+-}(\delta\eta, \eta_w) - N_{++}(\delta\eta, \eta_w)}{N_+} + \frac{N_{-+}(\delta\eta, \eta_w) - N_{--}(\delta\eta, \eta_w)}{N_-} \right), \quad (13)$$

where the conditional probability $N_{+-}(\delta\eta, \eta_w)$ counts pairs with opposite charge which satisfy the criteria that their relative pseudorapidity $\delta\eta = \eta_+ - \eta_-$ in a given pseudorapidity window is η_w , ($\delta\eta \in \eta_w$), whereas $N_+(N_-)$ is the number of positive (negative) particles in the same interval. Similarly for N_{++} , N_{--} , and N_{-+} . The factor 1/2 ensures the normalization of $B(\delta\eta, \eta_w)$. All terms in Eq. (13) are calculated within PHSD using pairs from a given event and the resulting distributions are summed over all events.

Both balance-function and charge-fluctuation observables are generated from one-body and two-body observables which necessitates that they may be expressed in terms of spectra and two-particle correlation functions. The charge fluctuation is a global measure of the charge correlation and the balance function is a differential measure of the charge correlation; it therefore carries more information. Writing $N_{\pm} = \langle N_{\pm} \rangle_{\eta_w} + \delta N_{\pm}$, where $\langle \dots \rangle_{\eta_w}$ denotes the average in the phase-space region η_w , it is easy to show [60] that

$$\frac{\langle (Q_{ch} - \langle Q_{ch} \rangle)^2 \rangle}{\langle N_{ch} \rangle} \simeq 1 - \int_0^{\eta_w} d\delta\eta B(\delta\eta|\eta_w), \quad (14)$$

where $Q_{ch} = N_+ - N_-$ and $N_{ch} = N_+ + N_-$.

From this example, one can readily understand how balance functions identify balancing charges. For any positive charge, there exists only one negatively charged particle whose negative charge originates from the point at which the

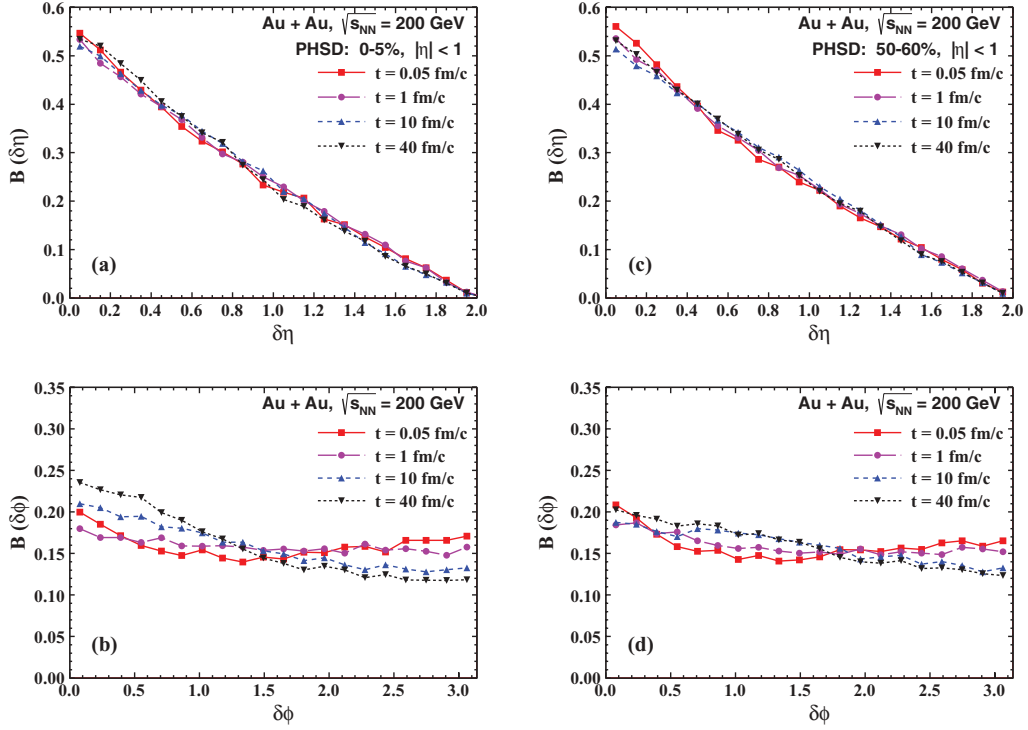


FIG. 15. (Color online) The balance function for charged quasiparticles (quark-antiquarks and \pm pions) with $|\eta_{+/-}| < 1$ from central (a),(b) and peripheral (c),(d) Au + Au ($\sqrt{s_{NN}} = 200$ GeV) collisions at $t = 0.05, 1, 10$, and 40 fm/c.

positive charge was created. By subtracting from the numerator the same object created with positive-positive pairs, one is effectively subtracting the uncorrelated contribution from the distribution.

It is expected that charge balance functions are sensitive to the separation of balancing charges in momentum space and give insight into the dynamics of hadronization [57]. Indeed, such a pair is composed of a positive and negative particle (or particle and antiparticle) whose charge originates from the same point in space-time. According to [57], if a quark-gluon plasma results in a large production of new charges (quark-antiquark pairs) late in the reaction, a tight correlation between the balancing charge-anticharge pairs would provide evidence for the creation of this novel state of matter.

The time evolution of the $\delta\eta$ - and $\delta\phi$ -dependent charge balance function is demonstrated in Fig. 15 for central [(a),(b)] and peripheral [(c),(d)] Au + Au collisions at $\sqrt{s_{NN}} = 200$ GeV. The times $t = 0.05$ and 1 fm/c correspond to the developed quark phase which ends at about $t = 10$ fm/c (cf. Fig. 4) while at $t = 40$ fm/c the system is in a purely hadronic phase. We recall that hadronization in the PHSD model is realized via a crossover transition and quasiparticle rescattering is included in both the partonic and hadronic phase.

As follows from Fig. 15, there is a very small difference in the time evolution of charge balance function $B(\delta\eta)$ at both centralities. As to $B(\delta\phi)$ a rather clear enhancement is seen for central collisions at $\delta\eta \sim \delta\phi \sim 0$ while this dependence is essentially weaker for peripheral collisions. This observation is in qualitative agreement with experiment but enhancement

effect is too small. Thus the expectation of a high sensitivity of the balance function to the hadron phase transition and particle diffusion seems somewhat too optimistic.

The direct comparison with experiment of the charge balance function is presented in Fig. 16 for Au + Au collisions at $\sqrt{s_{NN}} = 200$ GeV calculated within PHSD for central and peripheral ($b = 10$ fm) collisions. Here the charge conservation law is locally fulfilled in each quasiparticle collision. Hadronic resonance decays are taken into account in PHSD but corrections due to final state interactions for small $\delta\phi$ are computationally very involved (and uncertain).

As is seen from Fig. 16 there is a maximum for $\delta\eta \sim \delta\phi \sim 0$ but no large difference is observed for different centralities (apart from the $\delta\phi$ distribution). This finding is in agreement with the kinetic results of UrQMD and HIJING model calculations [51] of the width of the balance function in terms of $\delta\eta$ which also show no narrowing of the peak for central collisions as observed in experiments. The inclusion of the electromagnetic field [the dotted line in Fig. 16(b) for $b = 10$ fm] practically shows no influence and does not result in any additional focusing in central collisions.

We find that PHSD does not describe quantitatively the experimental balance functions. There are some claims that the blast-wave model can resolve this discrepancy [17,18,51]. The blast-wave model is in fact a parametrization of the kinetic freeze-out configuration motivated by a hydrodynamical model for the system described in local thermal equilibrium. The system is then completely characterized by the collective velocity profile, freeze-out temperature, and the freeze-out surface which is usually associated with some

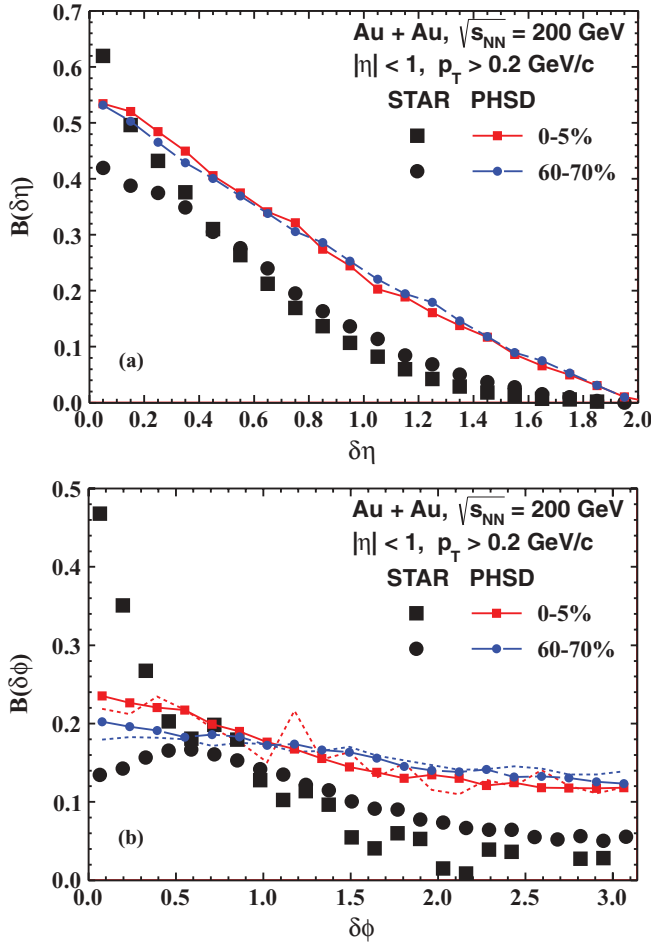


FIG. 16. (Color online) The balance function for pseudorapidity (a) and azimuthal angle (b) of charged pions with $|\eta_{+/-}| < 1$ and $p_T > 0.2$ GeV/c from central and peripheral Au + Au collisions at $\sqrt{s_{NN}} = 200$ GeV. The dotted lines for $\delta\phi$ correspond to calculations including the electromagnetic field effects. The experimental data points are from Ref. [51].

volume. Generally, the blast-wave model parameters may be varied in a large parameter space to fit experimental data. These single-particle freeze-out properties can, e.g., be parametrized as suggested in [61] to study, for example, the evolution of flow. However, the change in the kinetic freeze-out temperature and the increase of collective flow alone fail to explain the observed focusing of the balance function for more central collisions [18].

With regard to charge-balance correlations, the blast-wave model needs additionally to incorporate local charge conservation. This can be achieved in the following way: Instead of generating a single particle at a time, an ensemble of particles with exactly conserved charges is generated in such a way to remain unchanged the single-particle distributions. For the relative distribution of the pairs within an ensemble, a Gaussian distribution is assumed with dispersions σ_η^2 and σ_ϕ^2 for rapidity and transverse angle, respectively. Treating these dispersions as free parameters at every centrality it is possible to tune the narrowing effect for central events [18]. It is of interest that at the exactly central collision $\sigma_\eta^2 = \sigma_\phi^2 = 0$ and

they strongly grow with impact parameter reaching $\sigma_\eta^2 \approx 0.6$ and $\sigma_\phi^2/\pi \approx 0.4$ for centralities about 70% [18]. However, the additional assumption that balancing charges at the freeze-out are strongly correlated in momentum space is in conflict with the basic model assumption on thermodynamic equilibrium of the system.

In Ref. [18] the charge balance function was applied for the analysis of the CME. The charge separation between opposite-charge and same-charge two-pion correlators γ_P was defined as

$$\gamma_P \equiv \frac{1}{2}(2\gamma_{+-} - \gamma_{++} - \gamma_{--}) = \gamma_{+-} - \gamma_{ss}, \quad (15)$$

where the angle brackets in Eq. (7) include the balance function $B(p_+|p_-)$ as a weight factor for the balancing charges. The quantity γ_P can be estimated from available experimental data [9]. Since the PHSD is not successful in reproducing the charge balance function, there is not much sense to apply it for the charge separation γ_P . As follows from the comparison between the STAR data and the blast-wave model (including correlations and rescaling) results in reproducing the experimental normalization; the charge balance correlations for the relativistic charge separation are of the same size as the experimental signal and exhibit a similar qualitative behavior with respect to the centrality dependence [18]. The authors of Ref. [18] claim that their results are solid on the level of 10–20 %. The calculation of uncertainties originates predominantly from the particular parametrization of both the blast-wave model itself and, in particular, of the centrality dependence of the charge separation $B(p_a|p_b)$ in the azimuthal angle. However, the considered breakup physics differs significantly from more realistic scenarios as has been shown recently in Ref. [62]; the freeze-out temperature and baryon chemical potential—defining the chemical composition of the system—noticeably depend on centrality. Furthermore, there is some inconsistency in using a reaction-plane independent fit of the balance function for the CME signal where azimuthal angles are measured with respect to the reaction plane. It is also unclear how the extracted parameters change with the collision energy, however, the first preliminary STAR data on the collision-energy dependence of the balance function have been published recently [63]. Thus, a further careful study of this issue is needed.

IV. THE CME OBSERVABLE

The experimental signal of the possible CME is the azimuthal angle correlator calculated according to Eq. (7). The experimental acceptance $|\eta| < 1$ and $0.20 < p_T < 2$ GeV has been also incorporated in the theoretical PHSD calculations. Note that the theoretical reaction plane is fixed exactly by the initial conditions rather than by a correlation with a third charged particle as in the experiment [12]. Thus, within PHSD we calculate the observable (7) as a function of the impact parameter b or the centrality of the nuclear collisions which should be considered as a background of the CME signal. A comparison of the measured angular correlator with result of calculations is presented in Fig. 17. We mention that the calculation of this correlation is a very CP time consuming process and the proper statistical error bars are shown in Fig. 17.

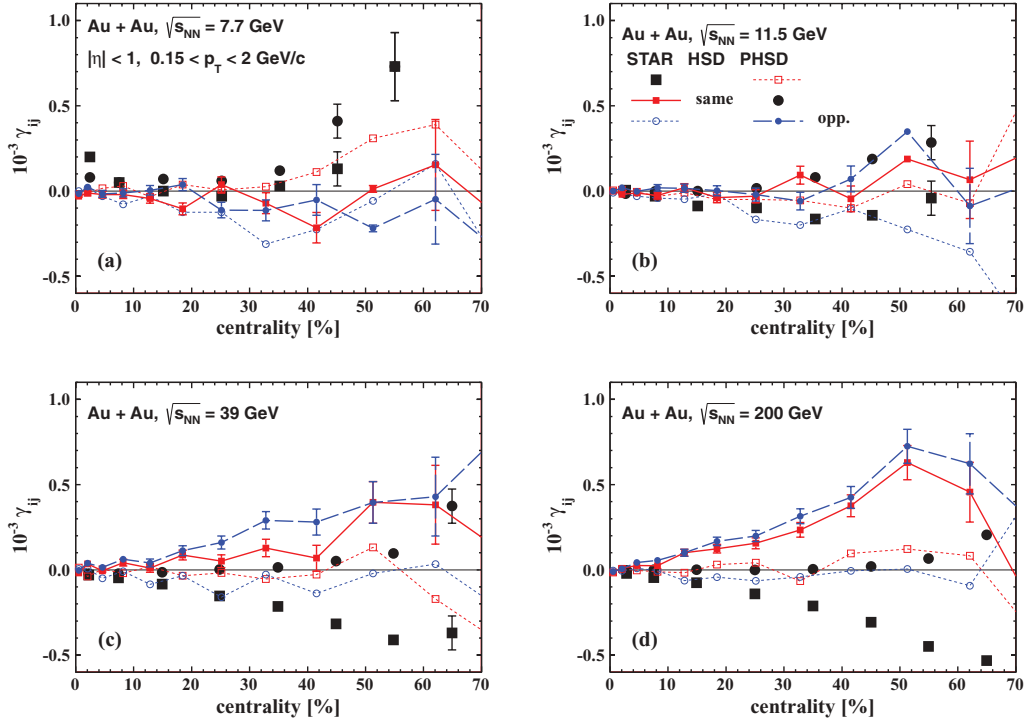


FIG. 17. (Color online) Angular correlations of opposite- and same-charge pions in azimuthal angle for Au + Au collisions at $\sqrt{s_{NN}} = 7.7$, 11.5, 39, and 200 GeV as a function of centrality. The full symbols are preliminary STAR data [12] as well as published STAR data for $\sqrt{s_{NN}} = 200$ GeV [9].

At the lowest measured energy $\sqrt{s_{NN}} = 7.7$ GeV the results for oppositely and same-charged pions are very close to each other and show some enhancement in very peripheral collisions. The centrality distributions of γ_{ij} are reasonably reproduced by the PHSD and HSD calculations presented in the same picture. Note that the scalar quark potential is not zero at this low energy but absent in the HSD model. The striking result is that the case of $\sqrt{s_{NN}} = 7.7$ GeV drastically differs from $\sqrt{s_{NN}} = 200$ GeV [cf. panels (a) and (d) in Fig. 17]. The picture quantitatively changes only slightly when one proceeds to $\sqrt{s_{NN}} = 11.5$ GeV [see panel (b) in Fig. 17] though the value of γ_{ij} at the maximum (centrality 70%) decreases a little bit in the calculations. Experimental points at larger centrality are not available but are of great interest. In addition, one may indicate a weak charge separation effect in the experimental data because statistical error bars are very small (less than the symbol size). Unfortunately, the calculated error bars are rather large to specify the charge separation effect. The influence of the electromagnetic field here is negligible. The calculated and measured correlation functions for oppositely and same charged pions are shown in Fig. 17 for the available three BES energies. The case for the top RHIC energy $\sqrt{s_{NN}} = 200$ GeV is also presented for comparison.

If one looks now at the results for $\sqrt{s_{NN}} = 39$ GeV, the measured same- and oppositely charged pion lines are clearly separated, being positive for the same-charged and negative for the oppositely charged pions to be strongly suppressed. The PHSD model is not able to describe this picture and overestimates the data with increasing energy. These growing large values of γ_{ij} are due to the scalar parton potential

which increases with the collision energy. The HSD version predicts a very small effect in qualitative agreement with our earlier analysis [26]. Though both models provide the charge separation essentially smaller than the measured one, the PHSD has a satisfying feature: the same-charge points are above the oppositely charged ones to be in agreement with experiment. The same situation is observed in the case of $\sqrt{s_{NN}} = 200$ GeV; a small difference between them is seen in very peripheral collisions: the oppositely charged correlation jumps to zero at centrality $\sim 70\%$ for $\sqrt{s_{NN}} = 39$ GeV while corresponding data at 200 GeV are not available.

Though the results at $\sqrt{s_{NN}} = 7.7$ and 11.5 GeV roughly can be considered as a background of the CME, at higher energies it is impossible to identify the true effect of the local parity violation as the difference between measured and PHSD results. The PHSD model [31] includes directly the dynamics of quark-gluon degrees of freedom which are becoming more important with increasing energy. We recall that the growing importance of the repulsive partonic mean field—illustrated earlier by the rise of the elliptic flow explained convincingly in the PHSD model [38,39]—results here in an overestimation of the CME background.

In Fig. 18 the results for the average cosine of the difference in the azimuthal angles δ_{ij} are presented. The measured centrality dependence for the same charge pions is flat and practically consistent with zero while that for oppositely charged particles is a monotonic increasing function with impact parameter. However, the PHSD calculations clearly overestimate the experimental points [9]. We note in passing that the PHSD results for Au + Au collisions at the energy

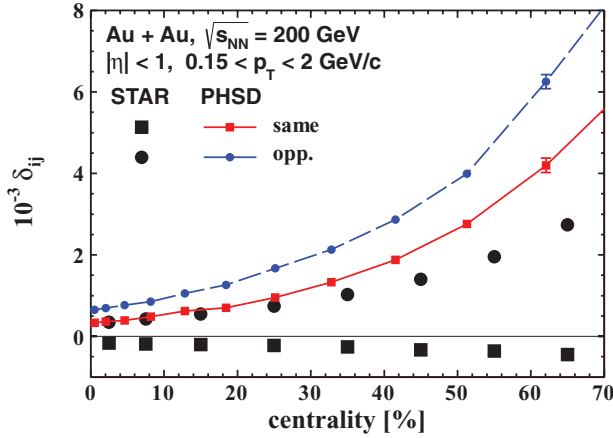


FIG. 18. (Color online) Angular correlations of opposite- and same-charge pions for the cosine of the difference in the azimuthal angles for Au + Au collisions at $\sqrt{s_{NN}} = 200$ GeV as a function of centrality. The experimental data points are from [9].

$\sqrt{s_{NN}} = 200$ GeV turn out to be astonishingly close to the appropriate experimental data at $\sqrt{s_{NN}} = 2.76$ TeV [13,14]. This fact indicates that the strength of the repulsive scalar quark potential in PHSD might be presently overestimated.

In accordance with Eq. (7), one can separate the in-plane and out-of-plane components using experimental results for γ_{ij} and δ_{ij} . Such separation together with PHSD calculation results is presented in Fig. 19 for the same charge and opposite charge pions. As was first noted in Ref. [16] and is seen in Fig. 19(a), for the same-charge pairs the sinus term is essentially zero whereas the cosine term is finite. This tells us that the observed correlations are actually in-plane rather than out-of-plane. This is contrary to the expectation from the chiral magnetic effect, which results in same-charge correlations out of plane. In addition, since the cosine term is negative, the in-plane correlations are stronger for back-to-back pairs than for small angle pairs. The PHSD does not reproduce these features. We see also that for opposite-charge pairs the in-plane and out-of-plane correlations are virtually identical. As was stated in [16], this is difficult to comprehend since there is a sizable elliptic flow in these collisions. Nevertheless, the PHSD model predicts very close in-plane and out-of-plane distributions for opposite-charge pairs due to scalar parton potential and at the same time nicely reproduces the various harmonics of charged particles [38,39]. This feature is not reproduced in the HSD.

We close this section with some more general remarks. As follows from the results presented in Figs. 17–19 an additional sizable source of asymmetry is needed for both in-plane and out-of-plane components rather than only an out-of-plane component as expected from the CME. As discussed in the Introduction, the vacuum nontrivial topological structure (as a genuine source of the CME) leads to the picture of a topological θ vacuum of non-Abelian gauge theories. The θ term in the QCD Lagrangian explicitly breaks \mathcal{P} and \mathcal{CP} symmetries of QCD. However, stringent limits on the value of $\theta < 3 \times 10^{-10}$ deduced from the experimental bounds on the electric dipole moment of the neutron [64] practically indicate the absence

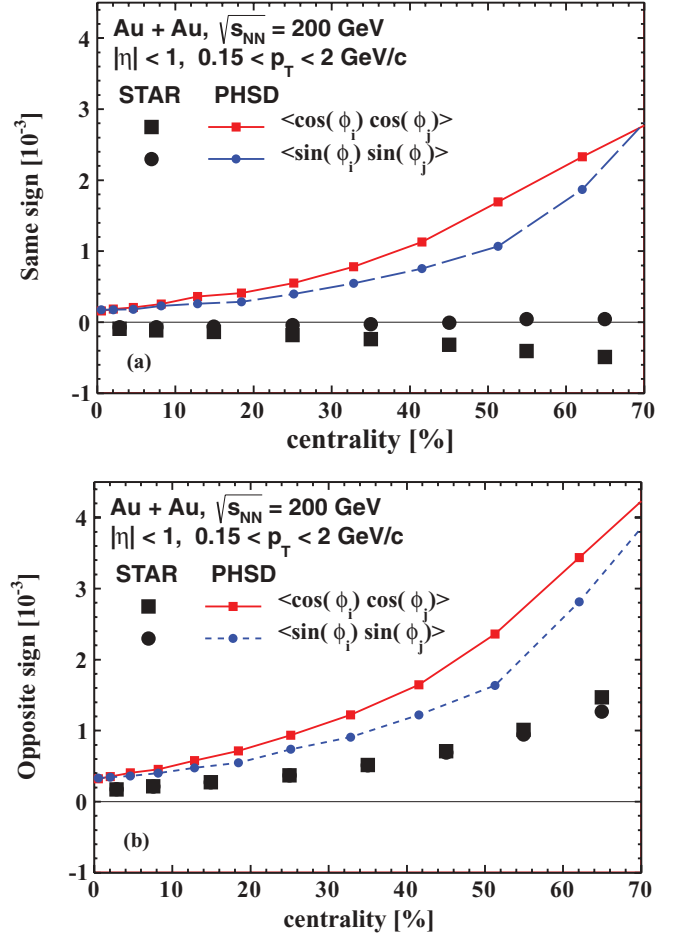


FIG. 19. (Color online) Angular correlations of $\cos(\text{out-of-plane})$ and $\sin(\text{in-plane})$ projections for Au + Au collisions at $\sqrt{s_{NN}} = 200$ GeV as a function of centrality. The experimental data points are from [9].

of *global* \mathcal{P} and \mathcal{CP} violation in QCD. Reference to the *local* \mathcal{P} - and \mathcal{CP} -odd effects due to the topological fluctuations characterized by an effective $\theta \equiv \theta(\mathbf{x}, t)$ varying in space and time [65] does not provide much hope. In addition, partons near the phase transition are not chiral (as typically assumed) but massive degrees of freedom in the PHSD in agreement with lattice QCD calculations. The finite mass of the partons washes out the chirality effect.

V. SUMMARY AND OUTLOOK

In this study we have investigated several effects that might contribute to the observed chiral magnetic effect (CME) in relativistic nucleus-nucleus collisions on the basis of event-by-event calculations within the PHSD transport approach. The individual results can be summarized as follows:

- (i) Our study shows that fluctuations in the position of quasiparticles can manifest themselves in different interaction stages and in different ways. Since the electromagnetic field generated by *spectators* is dominant at the early stage, the fluctuation in their position results in a noticeable fluctuation in the strength of the

electromagnetic field. However, the fluctuation spread is not so large as expected in the estimate from Ref. [29] and its influence on observables is negligible; in particular, the event plane angle is not tilted due to these electromagnetic field fluctuations.

- (ii) Early time fluctuations in the position of *participant* baryons were discussed in the past as a source of the impact parameter fluctuation. Its influence survives till the freeze-out resulting in a considerable difference between the theoretical reaction plane and the measured event plane. This effect leads to an increase in the magnitude of the elliptic flow and generates nonvanishing odd flow harmonics.
- (iii) We have found out that within the PHSD model the retarded strong electromagnetic field—created during nucleus-nucleus collisions—turns out to be not so important as has been expected before. Similarly to the HSD results in Ref. [7], the electromagnetic field has almost no influence on observables. The reason is not a shortness of the interaction time, when the electromagnetic field is maximal, but the compensation of the mutual action of transverse electric and magnetic components. This compensation effect might be important, for example, if an additional induced electric field (as a source of the CME) is available in the system since this field will not be entangled due to other electromagnetic sources.
- (iv) Another important point emerging from the compensation effect of electric and magnetic forces is worth mentioning: A significance of an external magnetic field in astrophysics is largely accepted. There are many studies where various effects of external magnetic fields are discussed in the application to astrophysics (e.g., see the Introduction in Ref. [7] and references in [66]). It is correct in this particular problems, however, in many cases it is concluded by a statement like “the same effect should be observed in high-energy heavy-ion collisions” which does not hold true due to the compensation effect as demonstrated in the present work.
- (v) In the intermediate stage of the heavy-ion collision the statistical fluctuations of charged quasiparticles in momentum space can generate charge dipoles or even charge quadrupoles. However, the magnitudes Q_{c1} and Q_{c2} are small; their orientation is distributed almost uniformly and the direction of the main axis is changed from event to event. The influence of the electromagnetic field here is negligible again.
- (vi) The transverse momentum conservation—proposed as an alternative mechanism for an explanation of the observed azimuthal asymmetry—shows a correlation of the CME and the elliptic flow. However, the effect estimated at $\sqrt{s_{NN}} = 200$ GeV is too small and insensitive to the charge separation.
- (vii) A possible charge separation of balancing charges has been addressed by the charge balance function. We note that the PHSD model fails to describe the focusing effect of the balance function for central Au + Au collisions. Certainly, further investigations of

this problem are needed, both in theory and experiment especially at lower energies.

The PHSD approach naturally takes into account the main alternative mechanisms of the CME: the momentum conservation and local charge conservation as well as clusters (mini jets, strings, prehadrons, resonances). At the moderate energies $\sqrt{s_{NN}} = 7.7$ and 11.5 GeV the PHSD model results are close to the experiment since partonic degrees-of-freedom are subleading. However, at higher collision energy the PHSD model fails to reproduce the observed azimuthal asymmetry. In contrast with our earlier analysis within the HSD model [26], the PHSD overestimates the measured centrality dependence of azimuthal distributions due to an increasing action of the repulsive scalar parton potential which generates the collective flow harmonics in accordance with experiment. This finding suggests that a new source of azimuthal anisotropy fluctuation is needed beyond the ‘standard’ interactions incorporated in PHSD. The new source does not dominate in out-of-plane direction as could be expected for the CME but both in-plane and out-of-plane components contribute with a comparable strength. In this respect the interpretation of the CME STAR measurements is still puzzling.

The present PHSD model is already quite elaborated, however, as our analysis has shown, color degrees of freedom or intimate peculiarities of non-Abelian Yang-Mills theory should additionally be taken into consideration. In particular, this concerns the very early stage of the nuclear interaction. In this initial state the highly compressed strongly interacting matter is dense and though the QCD coupling constant is small, gluonic states have high occupation numbers, i.e., the partons begin to overlap in phase space which leads to some saturated state. Strong color forces might create strong chromoelectric and chromomagnetic fields producing a new state, a *glasma* [44,46,67] as mentioned above in context of the discussion in Sec. III A, or forming new objects like “string ropes” described in the framework of Yang-Mills theory [68]. We are planning to include these effects into the PHSD model in near future.

Another class of strong fields relevant to the chirality and confinement of dynamical quarks is the long range (or soft) vacuum gluon field configurations. Long range vacuum gluon fields can be seen as an origin of nonzero gluon condensate and topological susceptibility of QCD vacuum [69,70]. Soft fields arise in the consideration of the global minima of the QCD effective action [71] and are known to play an important role in hadron phenomenology at zero temperature [72]. The nonzero gluon condensate survives at high temperature as demonstrated by QCD lattice calculations [73]. Interplay of strong electromagnetic and vacuum long-range gluon fields can lead to the qualitatively new effects in high energy heavy ion collisions [74]. However these effects are beyond the scope of this paper.

ACKNOWLEDGMENTS

We are thankful to Che Ming Ko, Sergei Molodtsov, Sergei Nedelko, Oleg Teryaev, Sergei Voloshin, and Harmen Warringa for illuminating discussions. This work has been supported by the LOEWE Center HIC for FAIR, a Heisenberg-Landau grant, RFFI Grant No. 11-02-01538-a, and by DFG.

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